

The Vertex Form of a Quadratic Function

Date: _____

VERTEX FORM**vs.****STANDARD FORM**

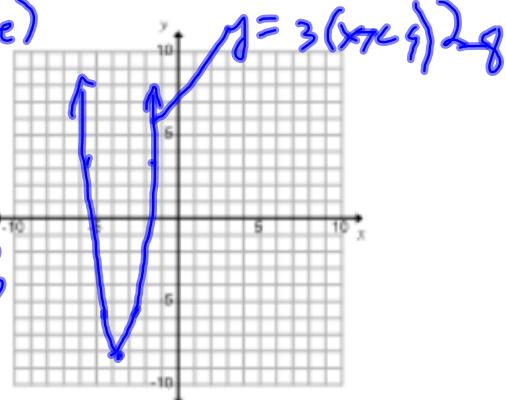
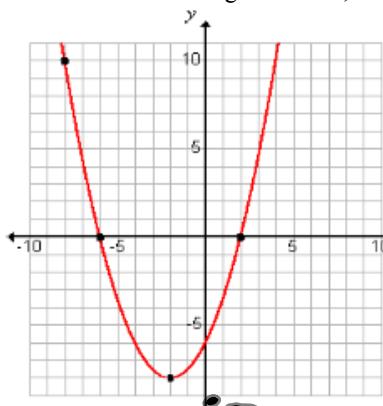
$$\begin{aligned} g(w) &= -(w-15)^2 + 225 \quad (\text{Show by expanding}) \\ &= -(w^2 - 30w + 225) + 225 \\ &= -w^2 + 30w - 225 + 225 \\ &= -w^2 + 30w \end{aligned}$$

Ex. 1: Given $f(x) = 3(x+4)^2 - 8$, determine:

- a) direction of opening *upwards (a is positive)*
 b) the axis of symmetry $x = -4 \quad x+4=0$
 c) the max/min value -8
 d) the vertex $(-4, -8)$
 e) State the Domain and Range $D = \{x \in \mathbb{R}\}$
 $R = \{y \in \mathbb{R} | y \geq -8\}$

$$f(w) = -w^2 + 30w$$

f) the sketch of the graph (no technology)

**Ex. 2:** Given the diagram below, write the quadratic equation in each of the 3 forms.

$$\begin{aligned} y &= a(x+2)^2 - 8 \\ 10 &= a(-6+2)^2 - 8 \\ 10 &= a(-4)^2 - 8 \\ 10+8 &= 16a \\ 18 &= 16a \\ \frac{18}{16} &= a \\ a &= \frac{9}{8} \end{aligned}$$

$$\begin{aligned} y &= a(x+6)(x-2) \\ 10 &= a(-8+6)(-8-2) \\ 10 &= a(-2)(-10) \\ 10 &= 20a \\ \frac{10}{20} &= a \\ a &= \frac{1}{2} \end{aligned}$$

$\therefore y = \frac{1}{2}(x+6)(x-2)$
is the equation in factored form

$$\begin{aligned} y &= \frac{1}{2}(x^2 + 4x - 12) \\ y &= \frac{1}{2}x^2 + 2x - 6 \end{aligned}$$

is the equation in standard form

$$\begin{aligned} y &= \frac{1}{2}(x^2 + 4x + 4) - 8 \\ &= \frac{1}{2}x^2 + 2x + 2 - 8 \\ &= \frac{1}{2}x^2 + 2x - 6 \end{aligned}$$

Ex. 3: What information about the parabola does each form provide?

Standard Form	Factored Form	Vertex Form
gives the y-intercept ("c" value)	gives the x-intercepts (zeros)	gives the vertex * the max/min value and when it occurs

Ex. 4: Write the equation of the quadratic function, first in vertex form and then in standard form.

a) vertex $(-6, 7)$ and passing through $(2, -9)$

$$\begin{aligned}
 y &= a(x+6)^2 + 7 \\
 (-9) &= a((2)+6)^2 + 7 \\
 -9 &= a(8)^2 + 7 \\
 -9 - 7 &= 64a \\
 -16 &= 64a \\
 \frac{-16}{64} &= a
 \end{aligned}
 \quad \left. \begin{aligned}
 a &= -\frac{1}{4} \\
 \therefore y &= -\frac{1}{4}(x+6)^2 + 7 \\
 &\text{in the eqn in vertex form.}
 \end{aligned} \right\}$$

$$\begin{aligned}
 y &= -\frac{1}{4}(x^2 + 12x + 36) + 7 \\
 &= -\frac{1}{4}x^2 - 3x - 9 + 7 \\
 &= -\frac{1}{4}x^2 - 3x - 2
 \end{aligned}$$