

Relating the Standard and Vertex Forms (by Completing the Square)

Date: Apr. 7/11**STANDARD FORM**

$$y = -8x^2 + 80x + 7$$

We are able to obtain "vertex form" by completing the square.

$$\begin{aligned} y &= -8(x^2 - 10x) + 7 \\ &= -8(x^2 - 10x + 25 - 25) + 7 \\ &= -8(x - 5)^2 + 200 + 7 \\ &= -8(x - 5)^2 + 207 \end{aligned}$$

vs.

VERTEX FORM

$$y = -8(x - 5)^2 + 207$$

Ex. 1 Write each quadratic function in vertex form by completing the square.

a) $y = x^2 + 8x + 10$

$$\begin{aligned} &= x^2 + 8x + 16 - 16 + 10 \\ &= (x + 4)^2 - 6 \end{aligned}$$

b) $f(x) = -4x^2 + 24x + 5$

$$\begin{aligned} &= -4(x^2 - 6x) + 5 \\ &= -4(x^2 - 6x + 9 - 9) + 5 \\ &= -4(x - 3)^2 + 36 + 5 \end{aligned}$$

Ex. 2 Write $y = 2x^2 - 3x - 4$ in vertex form.

$$\begin{aligned} y &= 2(x^2 - \frac{3}{2}x) - 4 \\ &= 2(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}) - 4 \\ &= 2(x - \frac{3}{4})^2 - \frac{9}{8} - \frac{32}{8} \\ &= 2(x - \frac{3}{4})^2 - \frac{41}{8} \end{aligned}$$

$$= -4(x - 3)^2 + 41$$

$$\begin{aligned} &\frac{1}{2}(\frac{3}{2}) \\ &= \frac{3}{4} \\ &\frac{2}{1}(\frac{-9}{16}) \\ &= -\frac{9}{8} \end{aligned}$$

Ex. 3 Judy wants to fence three sides of the yard in front of her house. She bought 100 m of fence and wants the maximum area she can fence in. The function $A(x) = 100x - 2x^2$ represents the area to be enclosed, where x is the width of the yard in metres.

a) Write the function in vertex form.

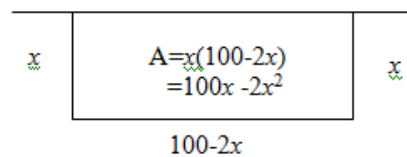
$$A(x) = 100x - 2x^2$$

$$= -2x^2 + 100x$$

$$= -2(x^2 - 50x)$$

$$= -2(x^2 - 50x + 625 - 625)$$

$$= -2(x - 25)^2 + 1250$$



b) Find the maximum area that can be enclosed.

the max. area
is 1250 m^2