Solutions 3

CHAPTER 3: Quadratic Models: Standard & Factored Forms

1. Write each of the following in standard form.
   (a) $f(x)=(3x+1)(x-2)$

   Standard form: $f(x)=3x^2-5x-2$

   (b) $f(x)=(2+3x)(x-3)$

   Standard form: $f(x)=2x-6+3x^2-9x$
   $=3x^2-7x-6$

2. Write each of the following in factored form.
   (a) $f(x)=x^2-16$
   $=(x-4)(x+4)$

   (b) $f(x)=x^2+3x-18$
   $=(x+6)(x-3)$

   (c) $f(x)=5x^2-20$
   $=5(x^2-4)$
   $=5(x-2)(x+2)$

3. Determine the zeros, the axis of symmetry, and the maximum and minimum value for each of the following quadratic equations. Show your work.

   (a) $f(x)=3x^2-3x$

   $f(x)=3x(x-1)$

   $\therefore x=0 \text{ and } x=1 \text{ are the zeros}$

   $\text{axis of symmetry}: x=\frac{1}{2}.$

   $f\left(\frac{1}{2}\right)=3\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)$

   $=3\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)$

   $=-\frac{3}{4}$

   $\therefore \min=-\frac{3}{4}$

   (b)

   $f(x)=-\frac{1}{2}(x^2+2x-3)$

   $=-\frac{1}{2}(x+3)(x-1)$

   $\therefore x=-3 \text{ and } x=1 \text{ are the zeros}$

   $\text{axis of symmetry}: x=-1$

   $f(-1)=-\frac{1}{2}((-1)^2+2(-1)-3)$

   $=-\frac{1}{2}(1-2-3)$

   $=-\frac{1}{2}(-4)$

   $=2$

   $\therefore \max=2$

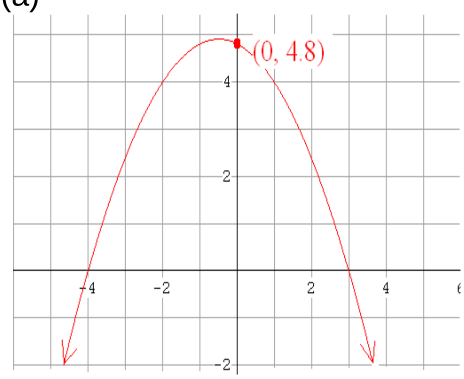
   (c) $f(x)=-4x^2-12x+7$

$$f(x) = -(4x^2 + 12x - 7)$$
$$= -(2x+7)(2x-1)$$
$$\therefore x = \frac{-7}{2} \text{ and } x = \frac{1}{2} \text{ are the zeros}$$

$$\text{axis of symmetry}: x = \frac{\frac{-7}{2} + \frac{1}{2}}{2}$$
$$x = \frac{\frac{-6}{2}}{2}$$
$$x = -\frac{3}{2}$$
$$f\left(-\frac{3}{2}\right) = -4\left(-\frac{3}{2}\right)^2 - 12\left(-\frac{3}{2}\right) + 7$$
$$= -4\left(\frac{9}{4}\right) + \frac{36}{2} + 7$$
$$= -9 + 18 + 7$$
$$\max = 16$$

4. Write the corresponding quadratic equation for each of the following functions.
Leave your answer in factored form.

(a)



$$y = a(x-r)(x-s)$$
$$= a(x+4)(x-3)$$
$$\text{sub in } (x, y) = (0, 4.8) \text{ to get}$$
$$4.8 = a(0+4)(0-3)$$
$$4.8 = a(-12)$$
$$\frac{4.8}{-12} = a$$
$$\frac{48}{-120} = a$$
$$\frac{2}{-5} = a$$
$$\therefore y = \frac{-2}{5}(x+4)(x-3)$$

(b)

The function has zeros at $x = 2$ and $x = 7$ and passes through the point $(0, -4)$

$$y = a(x-r)(x-s)$$
$$= a(x-2)(x-7)$$
$$\text{sub in } (x, y) = (0, -4) \text{ to get}$$
$$-4 = a(0-2)(0-7)$$
$$-4 = a(14)$$
$$\frac{-4}{14} = a$$
$$\frac{-2}{7} = a$$
$$\therefore y = \frac{-2}{7}(x-2)(x-7)$$

5. Can all quadratic equations be solved by factoring? Explain.

NO. Some quadratics do not pass through the x-axis….meaning there are NO zeroes.

6. Solve for x by factoring. Show your work.

(a) $4x^2 + 4x - 3 = 0$

(b) $x^2 + 6x - 3 = -3$

$$(2x+3)(2x-1)=0$$
$$\therefore x=\frac{-3}{2} \ \ and \ \ x=\frac{1}{2}$$

$$x^2+6x-3=-3$$
$$x^2+6x-3+3=0$$
$$x^2+6x=0$$
$$x(x+6)=0$$
$$\therefore x=0 \ \ and \ \ x=-6$$

7. A firecracker is fired from the ground. The height of the firecracker at a given time is modelled by the function $h(t)=-5t^2+40t$, where $h(t)$ is the height in metres and t is time in seconds.

(a) When will the firecracker hit the ground?

$$h(t)=-5t(t-8)$$
$$\therefore t=0 \ \ and \ \ t=8 \qquad \therefore it\ hits\ the\ ground\ after\ 8\ seconds.$$

(b) What is the maximum height of the firecracker?

$$h(t)=-5(t^2-8t+16)+80$$
$$=-5(t-4)^2+80$$
$$\therefore the\ \max=80\ metres$$

OR

$$axis\ of\ symmetry: x=4$$
$$h(4)=-5(4)^2+40(4)$$
$$=80$$

(c) When does the firecracker reach a maximum height?

$$the\ vertex=(4,80)$$
$$\therefore the\ \max\ occurs\ at\ 4\ seconds$$

(d) When will the firecracker reach a height of $75\ m$?

$$75=-5t^2+40t$$
$$0=-5t^2+40t-75$$
$$0=-5(t^2-8+15)$$
$$0=-5(t-3)(t-5)$$
$$\therefore t=3\ and\ t=5$$
$$\therefore the\ rocket\ reaches\ 75\,m\ at\ 3\ seconds\ (going\ up)$$
$$and\ at\ 5\ seconds\left(when\ the\ rocket\ is\ going\ down\right).$$

8. The population of a city $P(t)$ is modeled by the function $P(t)=0.5t^2+10t+200$, where $P(t)$ is the population in thousands and t is time in years. NOTE: $t=0$ represents the year 2000. According to the model,

(a) in what year will the population reach 312 000? *(see instructor for answer)*

(b) will the population reach over 2 million people by the year 2050? Show your work.

$$sub \ \ t=50$$
$$P(50)=0.5(50)^2+10(50)+200$$
$$=1950$$
$$So\ the\ population\ is\ 1950000$$
$$<2\ million$$
$$\therefore No.\ The\ population\ will\ not\ exceed\ 2\ million\ by\ 2050.$$

9. A quadratic equation has zeros $x = -4$ and $x = 2$. The minimum height is -5 units. Find the y-intercept for this quadratic equation (correct to 2 decimal places).

$$\frac{-4+2}{2} = \frac{-2}{2} = -1$$

$$\therefore vertex = (-1, -5)$$

$$y = a(x-r)(x-s)$$

$$y = a(x+4)(x-2)$$

$$-5 = a(-1+4)(-1-2)$$

$$-5 = a(3)(-3)$$

$$\frac{5}{9} = a$$

$$\therefore y = \frac{5}{9}(x+4)(x-2)$$

$$Let\ x = 0\ to\ find\ y-\text{int}ercept$$

$$y = \frac{5}{9}(0+4)(0-2)$$

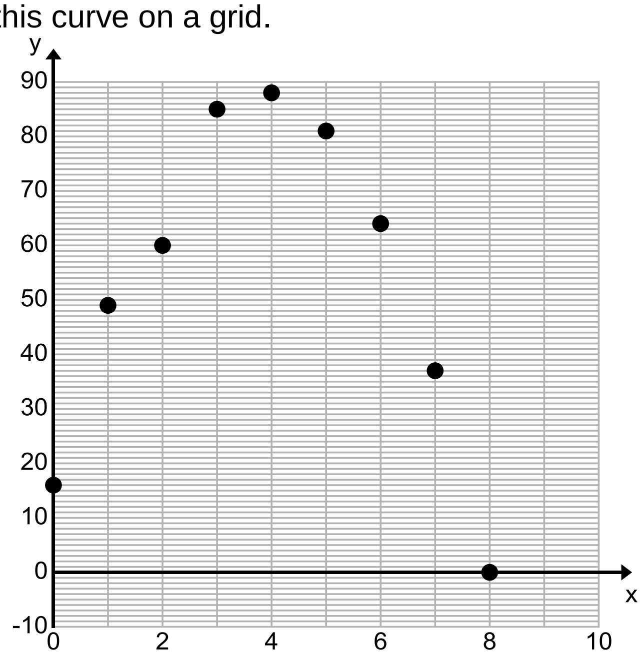
$$y = \frac{5}{9}(-8)$$

$$y = -4.44$$

10. A toy rocket sitting on a tower is launched vertically upward. Its height y at time t is given in the table.

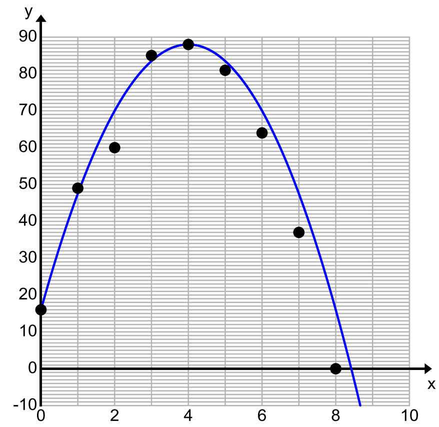
Time (in seconds)	***Height (in metres)***
0	16
1	49
2	60
3	85
4	88
5	81
6	64
7	37
8	0

(a) Sketch this curve on a grid.



(b) What is a possible equation for the curve of good fit? Show your work.

Let vertex be (4, 88) and use the y-intercept (0,16) to get



$y = a(x-h)^2 + k$

$sub\ vertex = (h,k) = (4,88)\ to\ get$

$y = a(x-4)^2 + 88$

$sub\ y-\text{int}\ which\ occurs\ at\ (0,16)$

$16 = a(0-4)^2 + 88$

$16 - 88 = a(16)$

$\frac{-72}{16} = a$

$-4.5 = a$

$\therefore y = -4.5(x-4)^2 + 88$