Solutions 4

CHAPTER 4: Quadratic Models: Standard & Factored Forms

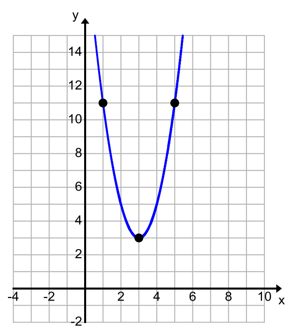
1. Write the function $f(x) = 2(x+3)^2 - 2$ in standard form.

$$\begin{aligned} f(x) &= 2(x+3)(x+3) - 2 \\ &= 2(x^2 + 6x + 9) - 2 \\ &= 2x^2 + 12x + 18 - 2 \\ &= 2x^2 + 12x + 16 \end{aligned}$$

2. For the function $f(x) = -(x-4)^2 + 1$, complete the table:

Vertex	(4, 1)
Axis of Symmetry	$x = 4$
Max/Min Value	max = 1
Domain	$\{x \in R\}$
Range	$\{y \in R \mid y \le 1\}$

3.

Determine the equation of the parabola . 	$y = a(x-h)^2 + k$ $y = a(x-3)^2 + 3$ $11 = a(1-3)^2 + 3$ $11 = a(-2)^2 + 3$ $11 - 3 = 4a$ $8 = 4a$ $2 = a$ $\therefore \quad y = 2(x-3)^2 + 3$

4. Write each function in vertex form and state the vertex.

(a) $f(x) = -x^2 + 6x + 7$

$$\begin{aligned} f(x) &= -(x^2 - 6x) + 7 \\ &= -(x^2 - 6x + 9 - 9) + 7 \\ &= -(x^2 - 6x + 9) + 9 + 7 \\ &= -(x-3)^2 + 16 \end{aligned}$$

$\therefore vertex = (3, 16)$

(b) $g(x) = 2x^2 - 3x + 3.5$

$$\begin{aligned} g(x) &= 2(x^2 - \frac{3}{2}x) + 3.5 \\ &= 2(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}) + 3.5 \\ &= 2(x^2 - \frac{3}{2}x + \frac{9}{16}) - \frac{18}{16} + 3.5 \\ &= 2(x - \frac{3}{4})^2 + 2.375 \end{aligned}$$

$\therefore vertex = (0.75,\ 2.375)$

5. The cost, $C(n)$, of operating a cement-mixing truck is modeled by the function $C(n)=2.2n^2-66n+700$, where n is the number of minutes the truck is running. What is the minimum cost of operating the truck? Show your work.

$$
\begin{aligned}
C(n) &= 2.2(n^2-30n \quad)+700 \\
&= 2.2(n^2-30n+225-225)+700 \\
&= 2.2(n^2-30n+225)-495+700 \\
&= 2.2(n-15)^2+205
\end{aligned}
$$

$$\therefore \min = 205$$

6. Solve using the quadratic formula. State your answers correct to 2 decimal places.

(a) $8x^2-6x+1=0$

$$
\begin{aligned}
&8x^2-6x+1=0 \\
&x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \\
&=\frac{6\pm\sqrt{(-6)^2-4(8)(1)}}{2(8)} \\
&=\frac{6\pm\sqrt{36-32}}{16} \\
&=\frac{6\pm\sqrt{4}}{16} \\
&\therefore x=\frac{6+2}{16} \quad \text{and} \quad x=\frac{6-2}{16} \\
&x=\frac{1}{2} \quad \text{and} \quad x=\frac{1}{4}
\end{aligned}
$$

(b) $x^2+3x=14$

$$
\begin{aligned}
&x^2+3x=14 \\
&x^2+3x-14=0 \\
&x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \\
&=\frac{-3\pm\sqrt{(3)^2-4(1)(-14)}}{2(1)} \\
&=\frac{-3\pm\sqrt{9+56}}{2} \\
&=\frac{-3\pm\sqrt{65}}{2} \\
&\therefore x=\frac{-3+\sqrt{65}}{2} \quad \text{and} \quad x=\frac{-3-\sqrt{65}}{2} \\
&x\doteq 2.53 \quad \text{and} \quad x\doteq -5.53
\end{aligned}
$$

7. A theatre company's profit can be modeled by the function $P(x)=-60x^2+700x-1000$ where x is the price of a ticket in dollars. What is the break-even price of the tickets?

$$
\begin{aligned}
&\text{Set } P(x)=0 \\
&0=-60x^2+700x-1000 \\
&a=-60,\ b=700,\ c=-1000
\end{aligned}
$$

$$
\begin{aligned}
x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \\
&=\frac{-700\pm\sqrt{700^2-4(-60)(-1000)}}{2(-60)} \\
&=\frac{-700\pm\sqrt{250000}}{-120} \\
&=\frac{-700\pm 500}{-120} \\
\therefore x&=\frac{-700+500}{-120} \quad \text{and} \quad x=\frac{-700-500}{-120} \\
&\doteq 1.67 \qquad\qquad\qquad =10.0
\end{aligned}
$$

NOTE: A "break even" point means that you neither made money nor lost money. ie…. $P(x)=0$

The break even points occur when tickets are sold for $1.67 and $10.

8. A model rocket is launched into the air. Its height, $h(t)$, in metres after t seconds is $h(t) = -5t^2 + 40t + 2$.

(a) When is the rocket at a height of 62 m (correct to 2 decimal places)?

$$62 = -5t^2 + 40t + 2$$
$$0 = -5t^2 + 40t + 2 - 62$$
$$0 = -5t^2 + 40t - 60$$
$$0 = -5(t^2 - 8t + 12)$$
$$0 = -5(t-6)(t-2)$$
$$\therefore t = 6 \quad and \quad t = 2$$

The rocket reaches 62m at 2 seconds (going up) and at 6 seconds (coming back down).

(b) What is the height of the rocket after 6 seconds?

62 metres. (see part (a) above)

(c) What is the maximum height of the rocket?

$$h(t) = -5(t^2 - 8t \quad) + 2$$
$$= -5(t^2 - 8t + 16 - 16) + 2$$
$$= -5(t^2 - 8t + 16) + 80 + 2$$
$$= -5(t-4)^2 + 82$$

The maximum height of the rocket is 82 metres at 4 seconds.

9. Without solving, determine the number of solutions of each equation. Show your work for full marks.

(a) $x^2 - 5x + 9 = 0$

$$x^2 - 5x + 9 = 0$$
$$b^2 - 4ac = (-5)^2 - 4(1)(9)$$
$$= 25 - 36$$
$$= -11$$
$$< 0$$
$$\therefore ZERO \ real \ roots$$

(b) $3x^2 - 5x - 9 = 0$

$$3x^2 - 5x - 9 = 0$$
$$b^2 - 4ac = (-5)^2 - 4(3)(-9)$$
$$= 25 + 108$$
$$= 133$$
$$> 0$$
$$\therefore TWO \ real \ roots$$

(c) $16x^2 - 8x + 1 = 0$

$$16x^2 - 8x + 1 = 0$$
$$b^2 - 4ac = (-8)^2 - 4(16)(1)$$
$$= 64 - 64$$
$$= 0$$
$$\therefore ONE \ real \ root$$

10. For the function $f(x)=kx^2+8x+5$, what value(s) of k will have two distinct solutions.

Two distinct real solutions means that $b^2-4ac>0$

$$
\begin{aligned}
&f(x)=kx^2+8x+5\\
&a=k,\ b=8,\ c=5\\
&b^2-4ac>0\\
&(8)^2-4(k)(5)>0\\
&64-20k>0\\
&64>20k\\
&\frac{64}{20}>k\\
&\frac{16}{5}>k
\end{aligned}
$$

11. The function $f(x)=x^2+kx+k+8$ touches the x-axis once. What value(s) could k be?

This means that there is only ONE real root. This happens when $b^2-4ac=0$.

$$
\begin{aligned}
&f(x)=x^2+kx+k+8\\
&a=1,\ b=k,\ c=(k+8)\\
&b^2-4ac=0\\
&(k)^2-4(1)(k+8)=0\\
&k^2-4k-32=0\\
&(k-8)(k+4)=0\\
&\therefore k=8 \quad or \quad k=-4
\end{aligned}
$$