Chapter 7 Solutions

CHAPTER 7: Exponential Functions

1. Write as a single power. Express answers with positive exponents. DO NOT EVALUATE.

(a) $4^3 \times 4 \times 4^2$
$= 4^6$

(b) $5(5^3)$
$= 5^4$

(c) $\dfrac{(-4)^6(-4)^3}{\left((-4)^9\right)^2}$
$= \dfrac{(-4)^9}{(-4)^{18}}$
$= (-4)^{-9}$
$= \dfrac{1}{(-4)^9}$

(d) $\dfrac{3^4}{\left(3^2\right)^3}$
$= \dfrac{3^4}{3^6}$
$= \dfrac{1}{3^2}$

(e) $\dfrac{\left(20^{-1}\right)^8}{20^2 20^6}$
$= \dfrac{20^{-8}}{20^8}$
$= 20^{-16}$
$= \dfrac{1}{20^{16}}$

(f) $\left(\dfrac{1}{9}\right)^5\left(\dfrac{1}{9}\right)^{-3}$
$= \left(\dfrac{1}{9}\right)^2$
$= \dfrac{1}{9^2}$

2. Evaluate WITHOUT using a calculator.

(a) $256^{\frac{-5}{4}}$
$= \left(\sqrt[4]{256}\right)^{-5}$
$= 4^{-5}$
$= \dfrac{1}{4^5}$
$= \dfrac{1}{1024}$

(b) $\left(-\dfrac{1}{2}\right)^3 + 2^{-3}$
$= -\dfrac{1}{8} + \dfrac{1}{8}$
$= 0$

(c) $4^{-1} + 4^0 + 4^2$
$= \dfrac{1}{4} + 1 + 16$
$= 17.25$

(d) $16^{\frac{3}{2}}$
$= \sqrt{16}^{\,3}$
$= 4^3$
$= 64$

(e) $\left(\dfrac{27}{64}\right)^{\frac{-1}{3}}$
$= \left(\dfrac{64}{27}\right)^{\frac{1}{3}}$
$= \dfrac{\sqrt[3]{64}}{\sqrt[3]{27}}$
$= \dfrac{4}{3}$

(f) $\sqrt[5]{-32}$
$= -2$

3. Complete the table.

Exponential Form	***Radical Form***	***Evaluation of Expression***
$81^{\frac{1}{4}}$	$\sqrt[4]{81}$	3
$27^{\frac{4}{3}}$	$\sqrt[3]{27}^4$	81
$7776^{\frac{1}{5}}$	$\sqrt[5]{7776}$	6
$4096^{0.75}$	$\sqrt[4]{4096}^3$	512

4. Use your calculator to evaluate each expression. Express answers to two decimals.

(a) $256^{0.66}$ = 38.85 (b) $15^{\frac{-3}{2}}$ =0.02 (c) $\sqrt[11]{3.7}$ =1.13 (d) $\sqrt[4]{-99}$ not possible

5. Complete the table.

Function	***Exponential Growth or Decay?***	***Initial Value (y-intercept)***	***Growth/Decay rate***
$P(n) = 200(1-0.032)^n$	decay	200	3.2%
$A(x) = (2)^x$	growth	1	100%
$Q(x) = 0.85(0.77)^x$	decay	0.85	23%

6. Calculate finite differences to classify each function as linear, quadratic, exponential or none of those.

(a)

x	y	Δy	Δ(Δy)
-4	47		
		-21	
-3	26		6
		-15	
-2	11		6
		-9	
-1	2		6
		-3	
0	-1		

Conclusion __quadratic__________

(b)

x	y	Δy	Δ(Δy)
-1	0.125		
		0.125	
0	0.25		1.625
		1.75	
1	2		4.25
		6	
2	8		18
		24	
3	32		

Conclusion _____none__________

Formulas:

$$P = P_0(1+r)^n \qquad P = P_0(1-r)^n \qquad I(d) = I_0(1+r)^d \qquad N(d) = N_0(1+r)^d$$

7. Greg invests $750 in a bond that pays 4.3% per year.

(a) Calculate, to the nearest penny, what Greg's total amount will be after 4 years.

$A = 750(1.043)^4$

$A = 887.56 \qquad \therefore \textit{the amount will be } \$887.56$

(b) How much money did $750 earn in four years?

$887.56 - 750 = 137.56 \qquad \therefore \textit{it earned } \$137.56$

(c) If Greg is planning to enter University in 2018, would his money have doubled by then?

$2018 \text{ is } 8 \text{ years from now}$

$A = 750(1.043)^8$

$A = 1050.35 \quad \therefore \text{he did not get } \$1500, \text{ so his money did not double}$

8. A police diver is searching a harbour for stolen goods. The equation that models the intensity of light per metre of depth is $I(n) = 100(0.92)^n$.

(a) At what rate does the light diminish per metre?

$1 - r = 0.92$

$r = 0.08 \quad \therefore \text{the light diminishes by } 8\% \text{ per metre}$

(b) Determine the amount of sunlight the diver will have at a depth of 18 m, relative to the intensity at the surface.

$I(18) = 100(0.92)^{18}$

$I(18) = 22.29 \quad \therefore \text{the light is } 22.29\% \text{ as intense as it was at the surface}$

9. Ryan purchases a used vehicle for $11, 899. If the vehicle depreciates at a rate of 13% yearly, what will the car be worth, to the nearest dollar, in ten years? Show your work.

$P(10) = 11899(1 - 0.13)^{10}$

$P(10) = 2955.99 \quad \therefore \text{it will be worth } \$2955.99 \text{ in } 10 \text{ years}$

10. After being filled, a basketball loses 3.2% of its air every day. The initial amount of air in the ball was 840 cm^3

(a) Write an equation to model this situation.

Let $P(t)$ represent the final amount of air left in the ball after t days

$P(t) = 840(1 - 0.032)^t$

(b) Determine the volume after 4 days.

$P(4) = 840(0.968)^4$

$P(4) = 737.53 \quad \therefore \text{there is } 737.53 \ cm^3 \text{ of air left in the ball}$

(c) Will this model be valid after 6 weeks? Explain.

We can still use the equation for t = 42 (6 weeks = 42 days). However, the ball probably won't be losing air at the same rate (it will be losing it at a much slower rate that it was when it was first pumped up) so the equation will not model the situation accurately after so long.

11. List 4 characteristics of an exponential function.

Consider the function $f(x)=b^x$, b is positive and not equal to 1

- *domain is $\{x \in R\}$, range is $\{y \in R | y > 0\}$*
- *if b>1, the greater the value, the faster the growth*
- *if 0<b<1, the lesser the value, the faster the decay*
- *horizontal asymptote is y=0 (the x-axis)*
- *y-intercept is 1*

First and second differences are related by a multiplication pattern.

EXTRA QUESTIONS: Chapter 7 p. 526 # 1 - 8