

## Today's Learning Goal(s):

Date: \_\_\_\_\_

By the end of the class, I will be able to:

- a) prove trigonometric identities.

Last day's work:

## 5.5 Trigonometric Identities

Date: NOV. 26 /15

**Equations** are valid for only certain values of the variable.

For example:

$$2x + 1 = 7$$

$$x^2 - 5x - 14 = 0$$

*x = 3 is the  
only value*

$$(x-7)(x+2)$$

*∴ x = 7 and x = -2 only values  
to make statement  
true.*

**Identities** are valid for all **values** of the variable.

For example:

$$2(x + 3) = 2x + 6$$

$$x^2 + 6x + 9 = (x + 3)^2$$

Let's start with the circle definitions to develop some identities that we can use later.

**SYR CXR TYX**

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

Ex.1 Prove that  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

*Restriction:*

$LS = \tan \theta$

$$= \frac{y}{x}$$

$RS = \frac{\sin \theta}{\cos \theta}$

$$= \frac{\frac{y}{r}}{\frac{x}{r}}$$

$$= \frac{y}{r} \div \frac{x}{r}$$

$$= \frac{y}{r} \times \frac{r}{x}$$

$$= \frac{y}{x}$$

$\therefore LS = RS$

$\therefore QED.$

$\cos \theta \neq 0$

$\cos \theta = 0$

$\theta = \cos^{-1}(0)$

$= 90^\circ$

Ex.2 Prove that  $\sin^2 \theta + \cos^2 \theta = 1$

$LS = \sin^2 \theta + \cos^2 \theta$

$$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$

$$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$$

$$= \frac{y^2 + x^2}{r^2}$$

Recall:  $x^2 + y^2 = r^2$

$$= \frac{r^2}{r^2}$$

$$= 1$$

$RS = 1$

$(\sin \theta)^2$

 $= \sin^2 \theta$ 

$\therefore LS = RS$

$\therefore QED.$

Ex.3 Prove that      Use "known" identities.

$$\text{a) } \frac{\cos \alpha \tan \alpha}{\sin \alpha} = 1$$

$$\text{b) } \cos \phi = \frac{1}{\cos \phi} - \sin \phi \tan \phi$$

$$\begin{aligned}
 \text{LS} &= \frac{\cos \alpha \tan \alpha}{\sin \alpha} & \text{RS} &= 1 & \text{LS} &= \cos \phi & \text{RS} &= \frac{1}{\cos \phi} - \sin \phi \tan \phi \\
 &= \cancel{\cos \alpha} \left( \frac{\sin \alpha}{\cos \alpha} \right) & & & &= \frac{1}{\cos \phi} - \sin \phi \frac{\sin \phi}{\cos \phi} & & \\
 &= \frac{\sin \alpha}{\sin \alpha} & & & &= \frac{1}{\cos \phi} - \frac{\sin^2 \phi}{\cos \phi} & & \\
 &= 1 & & \therefore \cos^2 \theta + \sin^2 \theta = 1 & &= \frac{1 - \sin^2 \phi}{\cos \phi} & & \\
 && & \therefore \cos^2 \theta = 1 - \sin^2 \theta & &= \frac{\cos^2 \phi}{\cos \phi} & & \\
 && & \therefore \text{LS} = \text{RS} & & & & \\
 && & \therefore \text{Q.E.D.} & & & & \\
 && & & & & \therefore \text{LS} = \text{RS} = \cos \phi & \\
 && & & & & \therefore \text{QED.} &
 \end{aligned}$$

## Identities

### Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

### Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

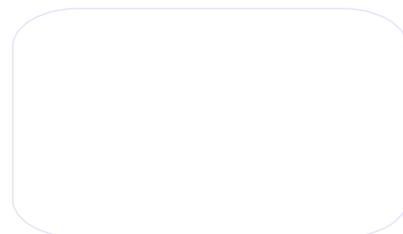
### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\begin{aligned}
 \sin^2 \theta + \cos^2 \theta &= 1 \\
 \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} &= 1 \\
 \tan^2 \theta + 1 &= \sec^2 \theta
 \end{aligned}$$



**Are there any Homework Questions you would like to see on the board?**

Last day's work:

Today's Homework Practice includes:

p. 310 #1 – 6

Note: Sometimes using substitution can help simplify a question.  
Ex. Simplify  $(1 - \cos\theta)(1 + \cos\theta)$       Change to  $(1 - a)(1 + a)$

$$\text{let } a = \cos\theta$$

$$\begin{aligned}
 &= 1 + \cos\theta - \cos\theta - \cos^2\theta &= 1 + a - a - a^2 \\
 &= 1 - \cos^2\theta &= 1 - a^2 \\
 &\text{but } a = \cos\theta &= 1 - \cos^2\theta \\
 &&= \sin^2\theta
 \end{aligned}$$