

## Today's Learning Goal(s):

Date: \_\_\_\_\_

By the end of the class, I will be able to:

- a) prove trigonometric identities.

Last day's work: p. 310 #1 – 6

## 5.5 Trigonometric Identities (Day2)

Date: \_\_\_\_\_

### Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

### Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

To Prove an Identity:

- \* Separate the LS and RS, and work on them separately
- \* convert *tan* and reciprocal ratios to *sin* or *cos*
- \* apply the Pythagorean Identity, use common denominators & factor as required

Ex.1 Prove that  $\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$

Ex.2 Prove that  $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = \frac{2}{\cos^2 \theta}$

## Are there any Homework Questions you would like to see on the board?

Last day's work: p. 310 #1 – 6

Today's Homework Practice includes:

pp. 310-311 #8, 10 – 12 [14]

Worksheet a – j

8. Prove each identity. State any restrictions on the variables.

a)  $\frac{\sin^2 \phi}{1 - \cos \phi} = 1 + \cos \phi$

b)  $\frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sin^2 \alpha$

c)  $\cos^2 x = (1 - \sin x)(1 + \sin x)$

d)  $\sin^2 \theta + 2 \cos^2 \theta - 1 = \cos^2 \theta$

e)  $\sin^4 \alpha - \cos^4 \alpha = \sin^2 \alpha - \cos^2 \alpha$

f)  $\tan \theta + \frac{1}{\tan \theta} = \frac{1}{\sin \theta \cos \theta}$

b) Solution 1: LS - use the quotient identity, and then simplify the fraction.

Solution 2: LS - use version 2 of the Pythagorean identity.

d) LS - sub in  $\sin^2 \theta$

e) LS - factor the difference of squares

f) LS - add the fractions then sub for  $\tan \theta$

12. Prove each identity. State any restrictions on the variables.

**1** a)  $\frac{\sin^2 \theta + 2 \cos \theta - 1}{\sin^2 \theta + 3 \cos \theta - 3} = \frac{\cos^2 \theta + \cos \theta}{-\sin^2 \theta}$

b)  $\sin^2 \alpha - \cos^2 \alpha - \tan^2 \alpha = \frac{2 \sin^2 \alpha - 2 \sin^4 \alpha - 1}{1 - \sin^2 \alpha}$

a) sub for  $\sin^2 \theta$  on both sides, then factor and divide.

b) LS - sub for  $\sin^2 \theta$  and use the quotient rule, then add the fractions.

### Extending

14. a) Which equations are not identities? Justify your answers.  
b) For those equations that are identities, state any restrictions on the variables.

i)  $(1 - \cos^2 x)(1 - \tan^2 x) = \frac{\sin^2 x - 2 \sin^4 x}{1 - \sin^2 x}$

ii)  $1 - 2 \cos^2 \phi = \sin^4 \phi - \cos^4 \phi$

iii)  $\frac{\sin \theta \tan \theta}{\sin \theta + \tan \theta} = \sin \theta \tan \theta$

iv)  $\frac{1 + 2 \sin \beta \cos \beta}{\sin \beta + \cos \beta} = \sin \beta + \cos \beta$

v)  $\frac{1 - \cos \beta}{\sin \beta} = \frac{\sin \beta}{1 + \cos \beta}$

vi)  $\frac{\sin x}{1 + \cos x} = \csc x - \cot x$

iv) LS - sub  $\sin^2 \theta + \cos^2 \theta$  in for 1, and then factor and divide.

v) LS - multiply top and bottom by  $1 + \cos \beta$

vi) RS - put in terms of  $\sin x$  and  $\cos x$  and then see above.