SEQUENCES AND SERIES REVIEW (WHITE BOARD)

Formulas to remember:

Arithmetic Sequence

General Term: $t_n = a + (n-1)d$

Recursive Formula: $t_n = t_{n-1} + d$, $n>1$

(n must be greater than 1) (n is a natural number)

Arithmetic series

$$S_n = \frac{n[2a+(n-1)d)]}{2} \qquad S_n = \frac{n}{2}[t_1 + t_n]$$

Geometric Sequence

General Term: $t_n = ar^{n-1}$

Recursive Formula: $t_n = rt_{n-1}$

Geometric Series

$$S_n = \frac{a(r^n - 1)}{r-1}$$

Pascal's Triangle: Patterns and application of binomial expansion

$$(a+b)^n$$

1. Determine if the following sequence is arithmetic, geometric or neither. Determine the general term for the sequence. Write the recursive formula.

a) 29, 21, 13, ...

$d = 21 - 29$
$= -8$ $\therefore$ a.seq.

$d = 29$

$t_n = a + (n-1)d$
$= 29 + (n-1)(-8)$
$= 29 - 8n + 8$
$= -8n + 37$

$t_1 = 29$
$t_n = t_{n-1} - 8$, $n>1$

i) determine t_{10}

$t_{10} = a + 9d$
$= 29 + 9(-8)$
$= 29 - 72$
$= -43$

b) 23, -46, 92, ...

$r = \frac{-46}{23}$
$= -2$

g.seq. $a = 23$

$t_n = ar^{n-1}$
$= 23(-2)^{n-1}$

$t_1 = 23$
$t_n = -2t_{n-1}$, $n>1$

i) determine t_{10}

$t_{10} = ar^9$
$= 23(-2)^9$
$= -11776$

2. Determine the general term for the arithmetic sequence if...

a) $t_1 = 13$ and $d = -7$

$= a$

$t_n = 13 + (n-1)(-7)$
$= 13 - 7n + 7$
$= -7n + 20$

b) $t_5 = 91$ and $t_7 = 57$

$t_5 = a + 4d$
$91 = a + 4d$

$57 = a + 6d$
$-91 = -a - 4d$
$-34 = 2d$
$d = -17$

$91 = a + 4(-17)$
$91 = a - 68$
$91 + 68 = a$
$159 = a$

$t_n = 159 + (n-1)(-17)$
$= 159 - 17n + 17$
$= -17n + 176$

3. Determine the general term for the geometric sequence if ...

a) the first term is 144 and the second term is 36

$a = 144$
$= t_1$

$t_2 = 36$

$r = \frac{36}{144}$
$= \frac{1}{4}$

$t_n = 144\left(\frac{1}{4}\right)^{n-1}$

b) $t_5 = 45$ and $t_8 = 360$

$ar^4 = 45$

$ar^7 = 360$

$\frac{\cancel{a}r^7}{\cancel{a}r^4} = \frac{360}{45}$

$r^3 = 8$

$\therefore r = 2$

$a(2)^4 = 45$

$a = \frac{45}{2^4}$
$= \frac{45}{16}$

$t_n = \frac{45}{16}(2)^{n-1}$

4. Calculate the sum of the first 10 terms in each series.

a) -103 -110 -117 - ...

$d = -110 - (-103)$
$= -7$
$\therefore$ a. ser.

$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_{10} = \frac{10}{2}[2(-103) + 9(-7)]$
$= -1345$

b) 8 -24 + 72 - ...

$r = \frac{-24}{8}$
$= -3$
$\therefore$ g. ser

$a = 8$

$S_n = \frac{a(r^n - 1)}{r - 1}$
$S_{10} = \frac{8((-3)^9 - 1)}{-3 - 1}$
$= \frac{8((-3)^9 - 1)}{-4}$
$= 39368$

5. Determine the sum of the first 7 terms of the geometric series if ...

a) the third term is 18 and the terms increase by a factor of 3

$t_3 = 18$ $r = 3$ $n = 7$

$ar^2 = 18$
$a(3)^2 = 18$
$a = 2$

$S_7 = \frac{2(3^7 - 1)}{3 - 1}$
$= \frac{\cancel{2}(3^7 - 1)}{\cancel{2}}$
$= 2186$

b) $t_5 = 5$ and $t_8 = -40$

6. Determine the sum of the first 7 terms of the arithemetic series if ...

a) $t_1 = 31$ and $t_{20} = 109$

$S_n = \frac{n}{2}[t_1 + t_n]$

b) $t_7 = 43$ and $t_{13} = 109$

$a + 6d = 43$

$$\begin{aligned} a + 12d &= 109 \\ -a - 6d &= -43 \\ \hline 6d &= 66 \\ d &= 11 \end{aligned}$$

$\therefore a + 6(11) = 43$

$a = 43 - 66$

$= -23 \leftarrow t_1$

$S_7 = \frac{7}{2}[t_1 + t_7]$

$= \frac{7}{2}[-23 + 43]$

$= 70$

7. Determine the number of terms in the sequence

-63, -57, -51, - ... +63

$d = -57 - (-63)$

$= +6$

$a = -63$

$t_n = a + (n-1)d$

$63 = -63 + (n-1)(6)$

$63 = -63 + 6n - 6$

$132 = 6n$

$n = 22$

$\therefore$ 22 terms in the seq.

8. Determine the sum of the geometric series.

17 - 51 + 153 - ... -334 611

9. At a fish hatchery, fish hatch at different times even though the eggs were all fertilized at the same time. The number of fish that hatched on each of the first four days after fertilization was 2, 10, 50, and 250, respectively. If the pattern continues, calculate the total number of fish hatched during the first 10 days.

10. Use Pascal's triangle to expand $(3x - 2y)^4$.

$(3x-2y)^4$

$= (3x)^4 + 4(3x)^3(-2y)^1 + 6(3x)^2(-2y)^2 + 4(3x)^1(-2y)^3 + 1(-2y)^4$

$= 81x^4 - 216x^3y + 216x^2y^2 - 96xy^3 + 16y^4$