MCR 3UI Ch. 7 Review

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a: $t_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[a + t_n]$

g: $t_n = ar^{n-1}$

$S_n = \frac{a(r^n - 1)}{r-1}$

3. i) general term ii) recursive formula

a) 58, 73, 88, ...
$a = 58$, $d = 15$
i) $t_n = 58 + (n-1)15$
$= 58 + 15n - 15$
$= 15n + 43$
ii) $t_1 = 58$, $t_n = t_{n-1} + 15$; $n > 1$

b) −49, −40, −31, ...
$a = -49$, $d = 9$
$t_n = -49 + (n-1)9$
$= -49 + 9n - 9$
$= 9n - 58$
$t_1 = -49$, $t_n = t_{n-1} + 9$, $n > 1$

c) 81, 75, 69, ...
$a = 81$, $d = -6$
$t_n = 81 + (n-1)(-6)$
$= 81 - 6n + 6$
$= -6n + 87$
ii) $t_1 = 81$, $t_n = t_{n-1} - 6$, $n > 1$

4. a. seq. t_{100} if $t_7 = 465$; $t_{13} = 219$
(∴ d = −ve)

$465 = a + 6d$
$219 = a + 12d$
$246 = -6d$
$d = -41$

$219 = a + 12d$
$219 = a + 12(-41)$
$219 + 492 = a$
$a = 711$

$t_{100} = 711 + 99(-41)$
$= -3348$

7 i) arith, geo, neither; ii) t_6

a) 5, 15, 45, ...
i) $r = 3$ ∴ geo
ii) $t_6 = 5(3)^5$
$= 1215$

b) 0, 3, 8, ...
i) neither

c) 288, 14.4, 0.72, ...
$r = 0.05$ ∴ geo
$t_6 = 288(0.05)^5$
$= 0.000\,09$

d) 10, 50, 90, ...
$d = 40$ ∴ arith
$t_6 = 10 + 5(40)$
$= 210$

e) 19, 10, 1, ...
$d = -9$ ∴ arith
$t_6 = 19 + 5(-9)$
$= -26$

f) 512, 384, 288, ...
$r = 0.75$
∴ geo.
$t_6 = 512(0.75)^5$
$= 121.5$

8. geo seq.; i) recursive formula ii) general term iii) first 5 terms

a) $a = 7$, $r = -3$
i) $t_1 = 7$, $t_n = -3t_{n-1}$, $n > 1$
ii) $t_n = 7(-3)^{n-1}$
iii) 7, −21, 63, −189, 567, ...

b) $a = 12$, $r = \frac{1}{2}$
$t_1 = 12$ $t_n = \frac{1}{2}t_{n-1}$, $n > 1$
$t_n = 12(\frac{1}{2})^{n-1}$
12, 6, 3, $\frac{3}{2}$, $\frac{3}{4}$, ...

c) $t_2 = 36$ $t_3 = 144$
$36 = ar$ $144 = ar^2$
$\frac{144}{36} = \frac{ar^2}{ar}$
∴ $r = \frac{144}{36} = 4$
∴ $a = 9$
i) $t_1 = 9$, $t_n = 4t_{n-1}$
ii) $t_n = 9(4)^{n-1}$ iii) 9, 36, 144, 576, 2304.

9 a) $t_n = 4n + 5$
arith.
9, 13, 17, 21, 25, ...

b) $t_n = \frac{1}{7n-3}$
neither
$\frac{1}{4}$, $\frac{1}{11}$, $\frac{1}{18}$, $\frac{1}{25}$, $\frac{1}{32}$, ...

c) $t_n = n^2 - 1$
neither
0, 3, 8, 15, 24, ...

d) $t_1 = -17$
$t_n = t_{n-1} + n - 1$, $n > 1$
−17, −16, −14, −11, −7, + ...
∴ neither

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10. doubles every hour
$\therefore r = 2$
$a = 23\,000$
if $t_1 = 1$ pm
then $t_{12} =$ midnight

a)
$t_n = ar^{n-1}$
$t_{12} = 23\,000(2)^{11}$
$= 47\,104\,000$

b) No; $\frac{1}{2}$ hour may be no new cells
ie) haven't split till late in hour
↳ Is there an unlimited food/oxygen supply to sustain infinite growth
↳ unlikely

14a) $1 + 9 + 17 + \ldots$
$a = 1 \quad d = 8$
$S_{50} = \frac{50}{2}[2(1) + 49(8)]$
$= 25(2 + 392)$
$= 9850$

c) $31 + 52 + 73$
$a = 31 \quad d = 21$
$S_{50} = \frac{50}{2}[2(31) + 49(21)]$
$= 25(62 + 1029)$
$= 27\,275$

e) $17.5 + 18.9 + 20.3 + \ldots$
$a = 17.5 \quad d = 1.4$
$S_{50} = \frac{50}{2}[2(17.5) + 49(1.4)]$
$= 25(35 + 68.6)$
$= 2590$

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15 S_{25} a. ser.

a) $a = 24 \quad d = 11$
$S_{25} = \frac{25}{2}[2(24) + 24(11)]$
$= 3900$

c) $t_1 = 84, \ t_2 = 57$
$a = 84, \ d = -27$
$S_{25} = \frac{25}{2}[2(84) + 24(-27)]$
$= -6000$

15e) $t_{12} = 19, \ d = -4$
$t_n = a + (n-1)d$
$t_{12} = a + 11d$
$19 = a + 11(-4)$
$a = 63$

$S_{25} = \frac{25}{2}[2(63) + 24(-4)]$
$= 375$

16a) $1 + 13 + 25 + \ldots + 145$
$a = 1 \quad d = 12$
$t_n = 1 + (n-1)(12)$
$145 = 12n - 11$
$156 = 12n$
$n = 13$
$S_{13} = \frac{13}{2}(1 + 145)$
$= 949$

c) $123 + 118 + 113 + \ldots - 122$
$a = 123 \quad d = -5$
$t_n = 123 + (n-1)(-5)$
$-122 = 123 - 5n + 5$
$-122 = -5n + 128$
$-250 = -5n$
$n = 50$

$S_n = \frac{n}{2}(2a + (n-1)d)$
$S_{50} = \frac{50}{2}[2(123) + 49(-5)]$
$= 25$

18 g. ser.; $t_6 = ?$, $S_6 = ?$

a) $11 + 33 + 99 + \ldots$
$a = 11 \quad r = 3$
$t_6 = 11(3)^{6-1}$
$= 11(3)^5$
$= 2673$
$S_6 = \frac{11(3^6 - 1)}{3 - 1}$
$= 4004$

18c) $6 - 12 + 24 - \ldots$
$a = 6 \quad r = -2$
$t_6 = 6(-2)^{6-1}$
$= 6(-2)^5$
$= -192$
$S_6 = \frac{6((-2)^6 - 1)}{-2 - 1}$
$= -126$

18e) $17 - 25.5 + 38.25 - \ldots$
$a = 17 \quad r = \frac{-25.5}{17}$
$= -1.5$ or $\frac{-3}{2}$

$t_6 = 17(-1.5)^{6-1}$
$= 17(-1.5)^5$
$= -129.09375$
or $t_6 = 17\left(\frac{-3}{2}\right)^5$
$= 17\left(\frac{-243}{32}\right)$
$= \frac{-4131}{32}$

$S_6 = \frac{17((-1.5)^6 - 1)}{-1.5 - 1}$
$= -70.65625$
or $S_6 = \frac{17\left[\left(\frac{-3}{2}\right)^6 - 1\right]}{\frac{-3}{2} - 1}$

$S_6 = 17\left(\frac{729}{64} - \frac{64}{64}\right) \div \left(\frac{-3}{2} - \frac{2}{2}\right)$
$= 17\left(\frac{665}{64}\right) \div \frac{-5}{2}$
$= 17\left(\frac{665}{64}\right) \times \frac{-2}{5}$
$= 17\left(\frac{133}{32}\right)$
$= \frac{-2261}{32}$

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19 g. Ser. ; $S_8 = ?$

b) $t_1 = 42$ $t_9 = 2112$

$\therefore a = 42$ $2112 = 42(r)^8$

$\frac{2112}{42} = r^8$

$\frac{352}{7} = r^8$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_8 = \frac{42\left(\frac{352}{7} - 1\right)}{\sqrt[8]{\frac{352}{7}} - 1}$

$= \frac{2070}{\sqrt[8]{\frac{352}{7}} - 1}$

$\doteq 3276.087344$

20.

$a = 15$ $r = 2$ $n = 12$

$S_{12} = \frac{15(2^{12} - 1)}{2-1}$

$= 61\,425$ orders filled at year's end

22. g. Series, $S_n = ?$

a) $7 + 14 + 28 + \ldots + 3584$

$a = 7$, $r = 2$

$3584 = 7(2)^{n-1}$

$512 = 2^{n-1}$

$\because 2^9 = 512$

$\therefore n = 10$

$S_{10} = \frac{7((2)^{10} - 1)}{2-1}$

$= 7161$

b) $-3 - 6 - 12 - 24 \ldots - 768$

$a = -3$ $r = 2$

$-768 = -3(2)^{n-1}$

$256 = 2^{n-1}$

$\because 2^8 = 256$

$\therefore n = 9$

$S_9 = \frac{-3((2)^9 - 1)}{2-1}$

$= -1533$

c) $1 + \frac{5}{2} + \frac{25}{4} + \ldots + \frac{15625}{64}$

$a = 1$ $r = \frac{5}{2}$

$\frac{15625}{64} = 1\left(\frac{5}{2}\right)^{n-1}$

$\because \left(\frac{5}{2}\right)^6 = \frac{15625}{64}$

$\therefore n = 7$

$S_7 = \frac{1\left(\left(\frac{5}{2}\right)^7 - 1\right)}{\frac{5}{2} - 1}$

$= \frac{\frac{78125}{128} - \frac{128}{128}}{\frac{3}{2}}$

$= \frac{77997}{128} \times \frac{2}{3}$

$= \frac{25999}{64}$

d) $96000 - 48000 + 24000 - \ldots + 375$

$a = 96000$ $r = -\frac{1}{2}$

$375 = 96000\left(-\frac{1}{2}\right)^{n-1}$

$\frac{1}{256} = \left(-\frac{1}{2}\right)^{n-1}$

$\because \left(-\frac{1}{2}\right)^8 = \frac{1}{256}$

$\therefore n = 9$

$S_9 = \frac{96000\left(\left(-\frac{1}{2}\right)^9 - 1\right)}{-\frac{1}{2} - 1}$

$= \frac{96000\left(\left(-\frac{1}{2}\right)^9 - 1\right)}{-\frac{3}{2}}$

$= 64\,125$

e) $1000 + 1000(1.06) + 1000(1.06)^2 + \ldots + 1000(1.06)^{12}$

$\therefore a = 1000$, $r = 1.06$ $n = 13$

$S_{13} = \frac{1000(1.06^{13} - 1)}{1.06 - 1}$

$\doteq 18\,882.137$

$\doteq 18\,882.14$

23a) $(a+6)^4$

$= a^4 + 4a^3(6) + 6a^2(6)^2 + 4a(6)^3 + 6^4$

$= a^4 + 24a^3 + 216a^2 + 864a + 1296$

c) $(2c+5)^3$

$= (2c)^3 + 3(2c)^2(5) + 3(2c)(5)^2 + 5^3$

$= 8c^3 + 60c^2 + 150c + 125$

e) $(5e - 2f)^4$

$= (5e)^4 + 4(5e)^3(-2f) + 6(5e)^2(2f)^2 + 4(5e)(-2f)^3 + (-2f)^4$

$= 625e^4 - 1000e^3f + 600e^2f^2 - 160ef^3 + 16f^4$