Before we begin, are there any questions from last day's work?

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) use a quadratic model to solve a problem with *and without* technology.

2.8.1 Modeling using Quadratic Functions

Date: Mar. 7/16

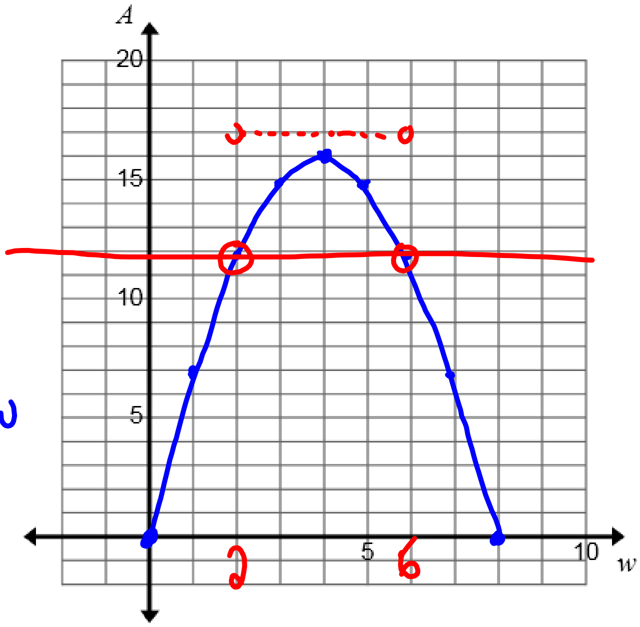
Ex.1
Sixteen metres of fencing are available to enclose a rectangular garden.
a) Represent the area of the garden as a function of the length of one side.
b) Graph the function.
c) What dimensions provide an area greater than 12 m^2?

Solution
a) Let w represent the width of the garden in m.
Let l represent the length of the garden in m.

l

$A = l \times w$

w



$P = 2l + 2w$
$16 = 2l + 2w$
$8 = l + w$
$8 - w = l$

$A = -w^2 + 8w$
$\therefore a = -1$

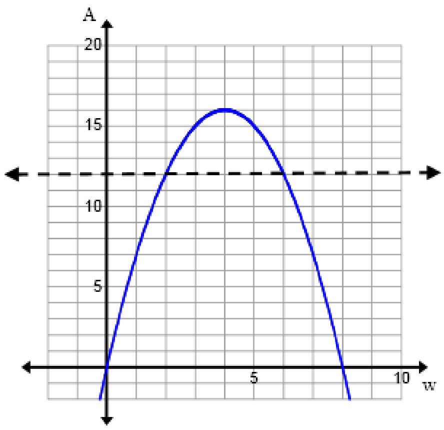
Since $A = lw$
$0 = (8 - w)w$

$8 - w = 0$ $w = 0$
$w = 8$

b)
the zeros (x-intercepts) are 0 and 8
Find the vertex half way between the zeros,
or complete the square to get $A = -1(w-4)^2 + 16$

vertex: $\frac{8+0}{2}$
$= 4$

Sub $w = 4$
$A = (8-4)(4)$
$= 16$
$\therefore (4, 16)$ is the vertex



c) **Draw in the horizontal line y = 12.**
The intersection points represent the width of the garden when the area is 12m².
∴if the width is between (but NOT INCLUDING) 2 and 6 m,
the dimensions provide an area greater than 12m².
This is written $2 < w < 6$

if c) asked $< 12\,m^2$
$\therefore 0 \le w < 2$ and $6 < w \le 8$

$$A = (8-w)w$$
$$= 8w - w^2$$
$$= -w^2 + 8w$$
$$= -(w^2 - 8w)$$
$$= -(\underbrace{w^2 - 8w + 16}-16)$$
$$= -(w-4)^2 + 16$$

$\therefore A = -(w-4)^2 + 16$

$\therefore V(4, 16)$

2.8.1 Modeling using Quadratic Functions

Ex. 2
When bicycles are sold for \$300 each, a cycle store can sell 160 in a season.
For every \$25 increase in the price, the number sold drops by 10.

a) Represent the sales revenue as a function of the price.
b) Use a graphing calculator to graph the function.
c) How many bicycles were sold when the total sales revenue is \$33 000? What is the price of one bicycle?
d) What range of prices will give sales revenue that exceeds \$40 000?

Solution
a) The quantities that vary all need to be defined (as variables).

Let p represent the selling price, in dollars.
Let n represent the number of bicycles sold.
Let R represent the revenue, in dollars.

Revenue = (price of a bicycle) x (number of bicycles sold)
p x (*needs to be represented as a function of price*)

(This is the hardest part of this problem.)

Rough work:

i) the price increase $= p - 300$

Check: If the new price is \$375, then the price increase $= p - 300$
$= 375 - 300$
$= 75$

ii) the number of \$25 increases $= \dfrac{p-300}{25}$

Check: If the new price is \$375, then the number of \$25 increases $= \dfrac{375-300}{25}$
$= \dfrac{75}{25}$
= 3 increases of \$25

iii) the number of bicycles sold

$$= 160 - \overset{2}{\cancel{10}}\left(\frac{p-300}{\underset{5}{\cancel{25}}}\right)$$

←Show Cancelling

$$= 160 - 2\left(\frac{p-300}{5}\right)$$

$$= 160 - \frac{2}{5}(p-300)$$

$$-\frac{2}{\cancel{5}}(-\overset{-60}{\cancel{300}}) = +120$$

$$= 160 - \frac{2}{5}p + 120$$

$$= -\frac{2}{5}p + 280$$

Now, Revenue = (price of a bicycle) x (number of bicycles sold)

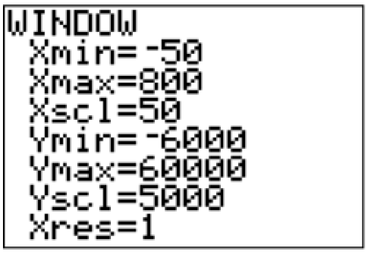
$$= p\left(-\frac{2}{5}p + 280\right)$$

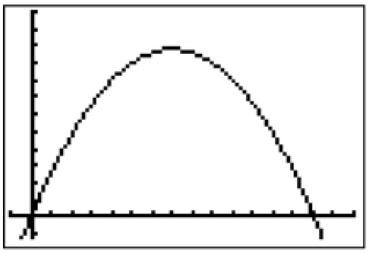
$$= -\frac{2}{5}p^2 + 280p$$

or $(= -0.4p^2 + 280p)$

b) Use a graphing calculator to graph the function.

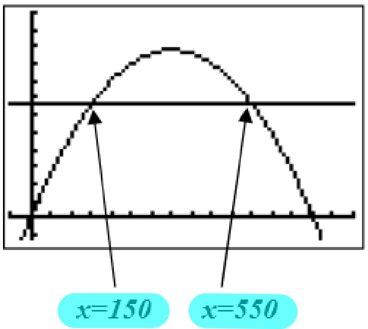
let $y_1 = -0.4x^2 + 280x$ or $y_1 = -\frac{2}{5}x^2 + 280x$





c) How many bicycles were sold when the total sales revenue is $33 000?
What is the price of one bicycle?

If $R = 33\ 000$, let $y_2 = 33000$



Find the intersection points to represent the price of one bicycle when the revenue is $33 000.

$\therefore p =$ **150** or $p =$ **550**

$\therefore$ the price of one bicycle is **$150** or **$550**

Recall: Revenue = (price of a bicycle) x (number of bicycles sold)

$\therefore$ number of bicycles sold $= \dfrac{\text{Revenue}}{\text{price of a bicycle}}$

if $p =$ **150**

number of bicycles $= \dfrac{33\ 000}{\mathbf{150}}$

$=$ **220**

or if $p =$ **550**

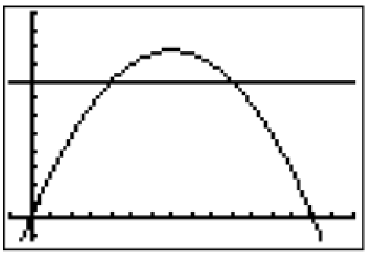
or number of bicycles $= \dfrac{33\ 000}{\mathbf{550}}$

or $=$ **60**

$\therefore$ **60** bicycles were sold if the sales revenue is $33 000.
(since the price *increases* will result in *lower* sales)

d) What range of prices will give sales revenue that exceeds $40 000?

If $R = 40\ 000$, let $y_3 = 40000$



(Don't forget to "turn off" y_2)

Find the intersection points to represent the price of one bicycle when the revenue is exactly $40 000.

$\therefore p =$ **200** or $p =$ **500**

Because we want when the revenue exceeds $40 000,
we DO NOT INCLUDE the intersection points in the solution.

$\therefore$ if $R > \$40\ 000$, then **200** $< p <$ **500**

Assigned Work pp. 224-225 #4(a-c), 6, 7, 10