Today's Learning Goal(s):

By the end of the class, I will be able to:

a) determine the centroid of any triangle.
b) show that the centroid divides the median in a 2:1 ratio.
c) show that joining the midpoints of any two sides creates a line segment that is parallel to the third side, and is half its length.

MPM 2DI 3.2 Verify Properties of Triangles Date: Mar. 21/16

To **verify** means to confirm, or demonstrate that something is true, accurate, or justified.

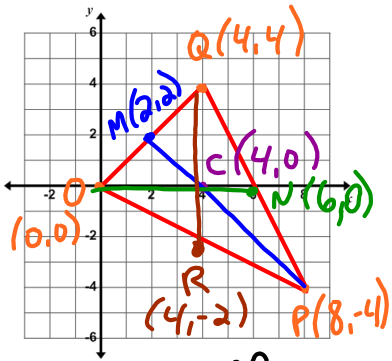
Recall: The centroid of a triangle is

the point where the three medians intersect.

Ex.1 Given: O(0, 0), P(8, -4) and Q(4, 4). Using analytic geometry, verify that:

a) C(4, 0) is the centroid of ΔOPQ.

b) the centroid divides each median in a 2:1 ratio.



Solution: Need Eq'n of Median from P

Let $M = M_{OQ}$

$M\left(\frac{0+4}{2}, \frac{0+4}{2}\right)$

$= M(2,2)$

$m_{MP} = \frac{-4-2}{8-2}$

$= \frac{-6}{6}$

$= -1$

$y = -1x + b$

$(2) = -1(2) + b$

$2 = -2 + b$

$2 + 2 = b$

$4 = b$

$\therefore y = -x + 4$ is the eq'n of the median from P

Verify C is on MP

LS = y = 0

RS = $-x + 4$ = $-(4) + 4$ = 0

$\therefore$ LS = RS

$\therefore$ C is on MP

N is the midpoint of QP

$N\left(\frac{4+8}{2}, \frac{4+(-4)}{2}\right)$

$= N\left(\frac{12}{2}, \frac{0}{2}\right)$

$= N(6,0)$

$m_{ON} = \frac{0-0}{6-0}$

$= \frac{0}{6}$

$= 0$

$\therefore y = 0x + b$

$(0) = 0(6) + b$

$0 = 0 + b$

$0 = b$

$\therefore y = 0x + 0$

$y = 0$ is the eq'n of the median from O.

$\underbrace{N(6,0) + O(0,0)}_{y=0}$ C(4,0)

Verify C is on ON

LS = y = 0

RS = 0

$\therefore$ LS = RS

$\therefore$ C is on ON

$R = M_{OP}$

$R\left(\frac{0+8}{2}, \frac{0+(-4)}{2}\right)$

$= R(4,-2)$

Eq'n of QR

$\underbrace{Q(4,4) + R(4,-2)}$

$x = 4$ is the eq'n of the median from Q.

Verify C is on QR

LS = x = 4

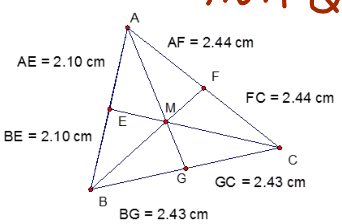
RS = 4

$\therefore$ LS = RS

$\therefore$ C is on QR

$\therefore$ C is the centroid

$*C(4,0)$



The centroid **always** divides each median into two parts where one length is twice that of the other length.

MPM 2DI 3.2 Verify Properties of Triangles Date: ____________________

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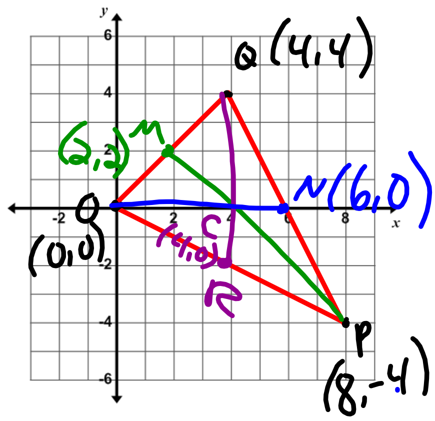
the point where the three medians intersect.

Ex.1 Given: O(0, 0), P(8, -4) and Q(4, 4). Using analytic geometry, verify that:

a) C(4, 0) is the centroid of $\triangle OPQ$.

b) the centroid divides each median in a 2:1 ratio.



Solution:

$$|CP| = \sqrt{(8-4)^2 + (-4-0)^2}$$
$$= \sqrt{(4)^2 + (-4)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32}$$
$$= \sqrt{16}\sqrt{2}$$
$$= 4\sqrt{2} \text{ units}$$

$$|CM| = \sqrt{(2-4)^2 + (2-0)^2}$$
$$= \sqrt{(-2)^2 + (2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= \sqrt{4}\sqrt{2}$$
$$= 2\sqrt{2} \text{ units}$$

Ratio of $\frac{|CP|}{|CM|}$

$$= \frac{4\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}}$$
$$= \frac{2}{1}$$
$$= 2:1$$

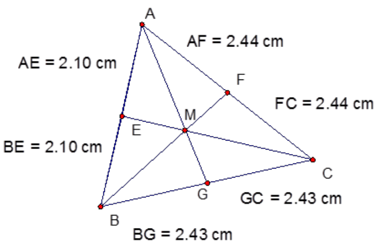
$|OC| = 4$ units $|CN| = 2$ units

$$\therefore \frac{|OC|}{|CN|} = \frac{4}{2}$$
$$= 2:1$$

$|CQ| = 4$

$|CR| = 2$

$$\therefore \frac{|CQ|}{|CR|} = \frac{4}{2}$$
$$= 2:1$$

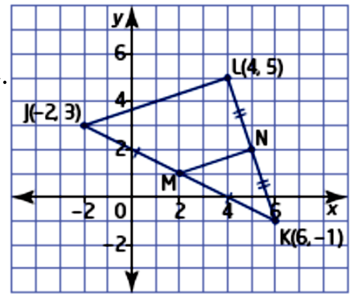


The centroid **always** divides each median into two parts where one length is twice that of the other length.

Ex.2 In $\triangle JKL$, M and N are midpoints.

a) Verify that line segment MN is parallel to JL.

b) Verify that line segment MN is half the length of JL.



Solution:

$M\left(\frac{-2+6}{2}, \frac{3+(-1)}{2}\right)$ | $N\left(\frac{4+6}{2}, \frac{5-1}{2}\right)$

$= M\left(\frac{4}{2}, \frac{2}{2}\right)$ | $= N\left(\frac{10}{2}, \frac{4}{2}\right)$

$= M(2, 1)$ | $= N(5, 2)$

$m_{MN} = \frac{2-1}{5-2}$

$= \frac{1}{3}$

$m_{JL} = \frac{5-3}{4-(-2)}$

$= \frac{2}{6}$

$= \frac{1}{3}$

$\therefore m_{MN} = m_{JL}$

$\therefore MN \parallel JL$

b) $|MN| = \sqrt{(5-2)^2 + (2-1)^2}$

$= \sqrt{3^2 + 1^2}$

$= \sqrt{10}$ units

$|JL| = \sqrt{(4-(-2))^2 + (5-3)^2}$

$= \sqrt{6^2 + 2^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

$= \sqrt{4}\sqrt{10}$

$= 2\sqrt{10}$ units

$\frac{|MN|}{|JL|} = \frac{1\cancel{\sqrt{10}}}{2\cancel{\sqrt{10}}}$

$= \frac{1}{2}$

$|MN| = \frac{1}{2}|JL|$

$= \frac{1}{\cancel{2}}(\cancel{2}\sqrt{10})$

$= \sqrt{10}$

The line segment joining the midpoints of any two sides of a triangle is **always** parallel to the third side and half its length.

Today’s entertainment: pp. 124-125 #1a, 2ab, 3, 4, 9, 12

Enrichment: p.126 #18

Also, some student's files are not in the "Members Area" of the class account.