Today's Learning Goal(s):

By the end of the class, I will be able to:

a) use the formulas for lengths, midpoints, and slopes to verify properties of quadrilaterals.

MPM 2DI 3.4 Verify Properties of Quadrilaterals Date: Mar. 23/16

From last class on *The Geometer's Sketchpad*, you learned:

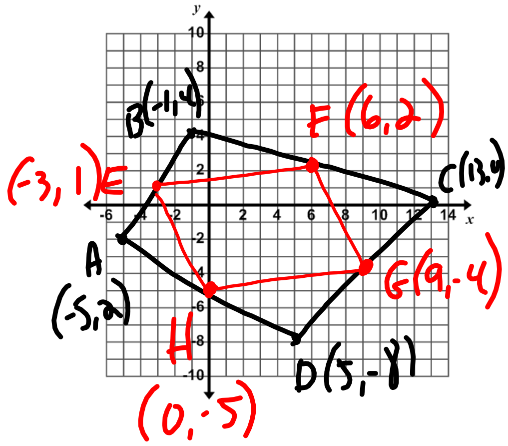
1. Joining the midpoints of adjacent sides of ANY quadrilateral forms a parallelogram.
2. In ANY parallelogram, the diagonals bisect each other.

To **verify** means to confirm, or demonstrate that something is true, accurate, or justified.

Ex.1 Given: A(-5, -2), B(-1, 4), C(13, 0) and D(5, -8). Using analytic geometry, verify that the quadrilateral formed by joining the midpoints of adjacent sides of quadrilateral ABCD, is a parallelogram.

Solution: **Discuss labelling in order: EFGH.**

- If the slopes of the opposite sides are equal, then EFGH is a parallelogram.
- First, calculate the midpoint of each side of ABCD.



$E = M_{AB}$

$E\left(\frac{-5+(-1)}{2}, \frac{-2+4}{2}\right)$

$E(-3, 1)$

$F = M_{BC}$

$= \left(\frac{-1+13}{2}, \frac{4+0}{2}\right)$

$= (6, 2)$

$G = M_{CD}$

$= \left(\frac{13+5}{2}, \frac{0+(-8)}{2}\right)$

$= (9, -4)$

$H = M_{AD}$

$= \left(\frac{-5+5}{2}, \frac{-2+(-8)}{2}\right)$

$= (0, -5)$

- Use the coordinates of the midpoints to calculate the slope of each side of EFGH.

$m_{EF} = \frac{2-1}{6-(-3)}$

$= \frac{1}{9}$

$m_{HG} = \frac{-4-(-5)}{9-0}$

$= \frac{1}{9}$

$m_{EH} = \frac{-5-1}{0-(-3)}$

$= \frac{-6}{3}$

$= -2$

$m_{FG} = \frac{-4-2}{9-6}$

$= \frac{-6}{3}$

$= -2$

$\because m_{EF} = m_{GH}$ $\therefore EF \parallel GH$

$\because m_{FG} = m_{EH}$ $\therefore FG \parallel EH$

$\therefore$ *EFGH* is a parallelogram.

Ex.2 Given: T(1, -2), U(3, 4), V(9, 6) and W(7, 0). Using analytic geometry:

a) Verify that the quadrilateral is a rhombus.
Sketch the rhombus.

b) Verify that the diagonals of TUVW bisect each other.

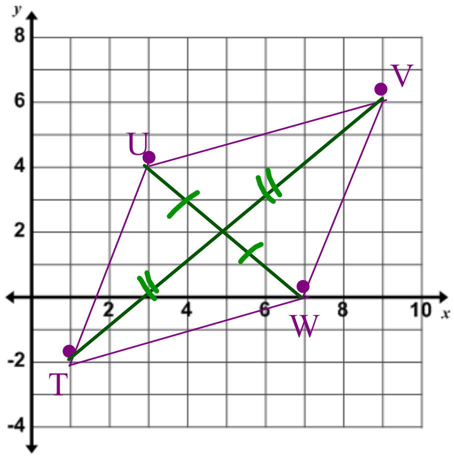
c) Verify that the diagonals of TUVW bisect each other at right angles.

Solution:

a) If all four sides are equal in length, then TUVW is a rhombus.

$|TU|$
$= \sqrt{(3-1)^2+(4-(-2))^2}$
$= \sqrt{(2)^2+(6)^2}$
$= \sqrt{4+36}$
$= \sqrt{40}$
$= \sqrt{4}\sqrt{10}$
$= 2\sqrt{10}$ units

$|UV|$
$= \sqrt{(9-3)^2+(6-4)^2}$
$= \sqrt{6^2+2^2}$
$= \sqrt{36+4}$
$= \sqrt{40}$
$= 2\sqrt{10}$ units



$|VW| = \sqrt{(7-9)^2+(0-6)^2}$
$= \sqrt{(-2)^2+(-6)^2}$
$= \sqrt{4+36}$
$= \sqrt{40}$
$= 2\sqrt{10}$ units

$|WT| = \sqrt{(1-7)^2+(-2-0)^2}$
$= \sqrt{(-6)^2+(-2)^2}$
$= \sqrt{36+4}$
$= \sqrt{40}$
$= 2\sqrt{10}$ units

∵ all 4 sides are equal in length
∴ TUVW is a rhombus

b) If the diagonals have the same midpoint, they bisect each other.

We could measure the length of each part of the diagonal, to see if they are equal. (but that would be more work).

Midpoint of **TV**.

$$\left(\frac{1+9}{2}, \frac{-2+6}{2}\right)$$
$$= \left(\frac{10}{2}, \frac{4}{2}\right)$$
$$= (5, 2)$$

Midpoint of **UW**.

$$\left(\frac{3+7}{2}, \frac{4+0}{2}\right)$$
$$= \left(\frac{10}{2}, \frac{4}{2}\right)$$
$$= (5, 2)$$

∵ the midpoints of the diagonals have the same coordinates,
∴ the diagonals bisect each other.

c) If the slopes of the diagonals are negative reciprocals, then the diagonals bisect each other at right angles.

$$m_{TV} = \frac{6-(-2)}{9-1} = \frac{8}{8} = 1$$

$$m_{uw} = \frac{0-4}{7-3} = \frac{-4}{4} = -1$$

Check $1(-1) = -1$

if product $= -1$

∵ the slopes are negative reciprocals
∴ TV ⊥ UW
∴ the diagonals bisect each other at right angles

Today's entertainment: pp. 142-143 #2, 4, 5, 10, 12, 14

Enrichment: p. 144 #17

Remember to begin working ahead on the Review: pp.152-155