Before we begin, are there any questions from last day's work?

Worksheet #1 to 6 *(#3,4,6 on next slides)*

***Enrichment*:** #7 to 10

Today's Learning Goal(s):

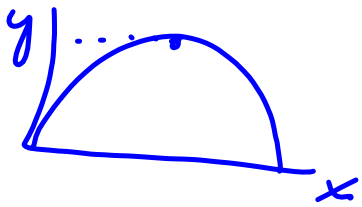
By the end of the class, I will be able to:

a) solve a quadratic equation that ***cannot*** be factored.

***Stress what completing the square does, and when to use it.**

3. An architect has designed a modern building that is to be supported by a steel arch shaped like a parabola. This parabola can be modelled by the relation $y = -0.025x^2 + 2x$, where y represents the height of the arch and x represents the distance along the base, both in metres. What is the highest point on the parabolic arch?

(Answer: 40 metres)



$y = -0.025x^2 + 2x$

$= -0.025(x^2 - 80x)$

$= -0.025(x^2 - 80x + 1600 - 1600)$

$= -0.025(x - 40)^2 + 40$

∴ the highest point is 40m.

4. Hiroshi is trying out for the position of kicker on the football team. He wants to know at what angle he should kick the ball for maximum distance. He has used a machine that kicks footballs with constant velocity but at varying angles. Hiroshi has collected some data and used quadratic regression on his graphing calculator to determine that the relation between angle and distance is given by the equation $d = -0.1a^2 + 8.5a - 40$, where a is the angle in degrees, and d is the distance in metres.

a) Determine the vertex of the parabola. *(Answer: (42.5, 140.625))*

b) Which angle gives the maximum distance? *(Answer: 42.5 degrees)*

c) For what values of a is the graph valid? *(Answer: $5 < a < 80$)*

a) $d = -0.1a^2 + 8.5a - 40$

$= -0.1(a^2 - 85a) - 40$

$= -0.1(a^2 - 85a + 42.5^2 - 42.5^2) - 40$

$= -0.1(a^2 - 85a + 1806.25 - 1806.25) - 40$

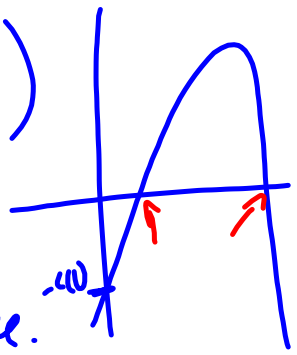
$= -0.1(a - 42.5)^2 + 180.625 - 40$

$= -0.1(a - 42.5)^2 + 140.625$

$\therefore V(42.5, 140.625)$

b) $\therefore$ a 42.5° angle gives maximum distance.

c)



c) $d = -0.1a^2 + 8.5a - 40$

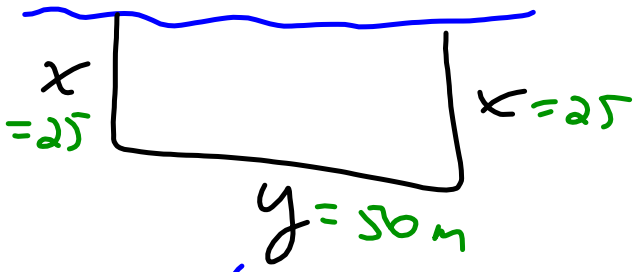
valid if $d > 0$

$0 = -0.1a^2 + 8.5a - 40$

$0 = -0.1(a^2 - 85a + 400)$

$= -0.1(a - 80)(a - 5)$

6. A field is bounded on one side by a river. The field is to be enclosed on three sides by a fence, to create a rectangular enclosure. The total length of fence to be used is 100 m. Use a quadratic model to determine the dimensions of the enclosure of maximum area. *(Answer: 25 m by 50 m)*



Perimeter (Fence) $= 2x + y$

$100 = 2x + y$

$100 - 2x = y$

$A = (l)(w)$

$A = xy$

$= x(100-2x)$

$= 100x - 2x^2$

$= -2x^2 + 100x$

$= -2(x^2 - 50x)$

$= -2(x^2 - 50x + 625 - 625)$

$= -2(x-25)^2 + 1250$

$\therefore$ if $x = 25$ the max. area is 1250

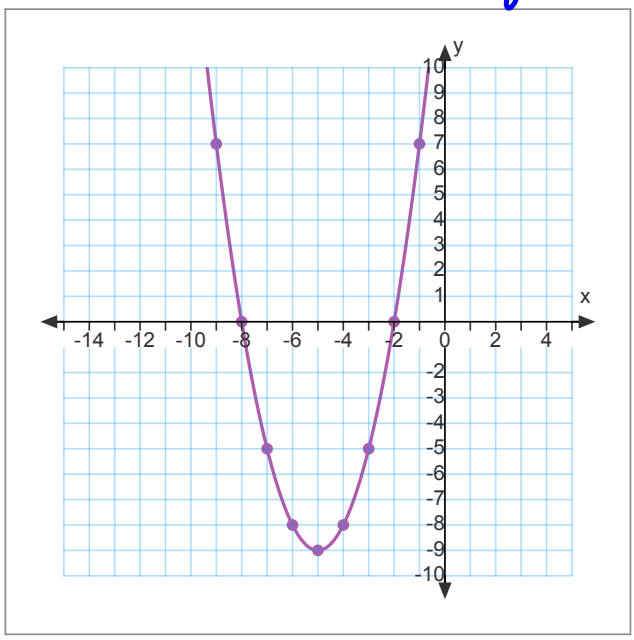
MPM 2DI 6.4 The Quadratic Formula (Day1) Date: May 13/16

Warm-up: Solve by factoring.

$x^2 + 10x + 16 = 0$

$(x+8)(x+2) = 0$

$\therefore x = -8$ or $x = -2$



Note: The related quadratic relation is:

$y = x^2 + 10x + 16$

If we complete the square to get vertex form:

$y = (x+5)^2 - 9$

$\therefore V(-5, -9)$

The solutions (or roots) for **any** quadratic equation

$$ax^2 + bx + c = 0$$

can be found using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Let's verify the formula using $x^2+10x+16=0$

$a=1 \quad b=10 \quad c=16$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(10)\pm\sqrt{(10)^2-4(1)(16)}}{2(1)}$$

$$=\frac{-10\pm\sqrt{100-64}}{2}$$

$$=\frac{-10\pm\sqrt{36}}{2}$$

$x=\frac{-10+\sqrt{36}}{2}$ or $x=\frac{-10-\sqrt{36}}{2}$

$=\frac{-10+6}{2}$ $=\frac{-10-6}{2}$

$=\frac{-4}{2}$ $=\frac{-16}{2}$

$=-2$ $=-8$

Ex.1 Solve.

$x^2+10x+17=0$ (This **clearly** does not factor!)

$a=1$ $b=10$ $c=17$

$$x=\frac{-(10)\pm\sqrt{(10)^2-4(1)(17)}}{2(1)}$$

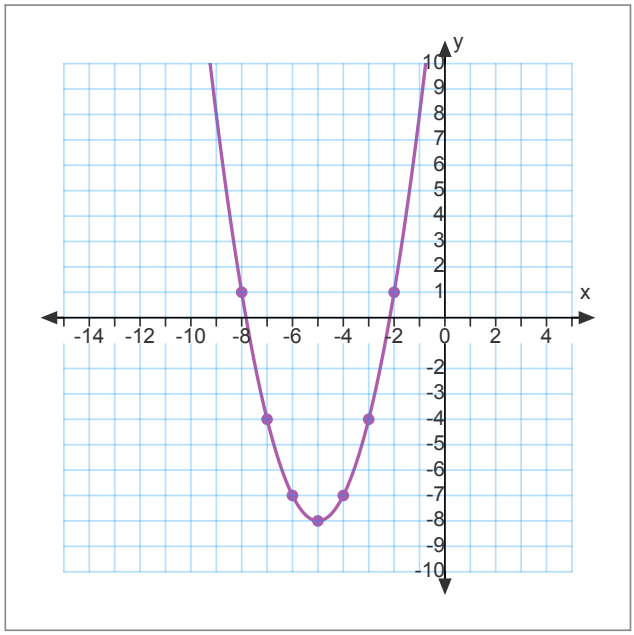
$$=\frac{-10\pm\sqrt{100-68}}{2}$$

$$=\frac{-10\pm\sqrt{32}}{2}$$

$$x=\frac{-10+\sqrt{32}}{2} \text{ or } x=\frac{-10-\sqrt{32}}{2}$$

$\doteq -2.1715$ $\doteq -7.8284$

$\doteq -2.172$ $\doteq -7.828$



Note: The related quadratic relation is:

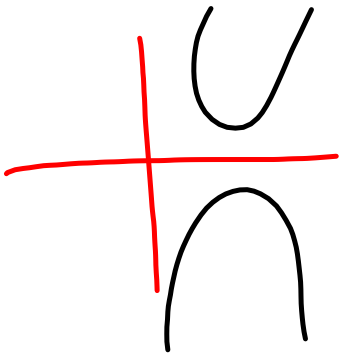
$$y=x^2+10x+17$$

If we complete the square to get vertex form:

$$y=(x+5)^2-8 \quad V(-5,-8)$$

This means you can now find the *x*-intercepts of any quadratic relation**...if they exist!**

When would they ***not*** **exist**?



Ex. 2 Use the quadratic formula to solve each equation.

Express your answers as exact roots **AND** as approximate roots, rounded to the nearest hundredth.
(without simplifying)

a) $2x^2+10x+3=0$

$a=2 \quad b=10 \quad c=3$

$$x=\frac{-(10)\pm\sqrt{(10)^2-4(2)(3)}}{2(2)}$$

$$=\frac{-10\pm\sqrt{100-24}}{4}$$

$$=\frac{-10\pm\sqrt{76}}{4}$$

$$x=\frac{-10+\sqrt{76}}{4} \text{ or } x=\frac{-10-\sqrt{76}}{4}$$

$\doteq 0.320$ $\doteq -4.679$

$\doteq 0.32$ $\doteq -4.68$

b) $-6x^2=7x-3$

$0=6x^2+7x-3$

$a=6 \quad b=7 \quad c=-3$

$$x=\frac{-(7)\pm\sqrt{(7)^2-4(6)(-3)}}{2(6)}$$

$$=\frac{-7\pm\sqrt{49+72}}{12}$$

$$=\frac{-7\pm\sqrt{121}}{12}$$

$$\therefore x=\frac{-7+\sqrt{121}}{12} \text{ or } x=\frac{-7-\sqrt{121}}{12}$$

$=\frac{-7+11}{12}$ $=\frac{-7-11}{12}$

$=\frac{4}{12}$ $=\frac{-18}{12}$

$=\frac{1}{3}$ $=\frac{-3}{2}$

Note: $6x^2+7x-3=0$

$(3x-1)(2x+3)=0$

Today's entertainment:

YOU **MUST** USE THE QUADRATIC FORMULA LEARNED TODAY.

p. 300 #1*, 2* * in the Answers section there are typos: whenever you come across a semi-colon (;) replace it with this symbol $\pm$

***Enrichment*:** p. 302 #16

How to Memorize the Formula?

Quadratic Formula Song (with equation)
http://www.youtube.com/watch?v=O8ezDEk3qCg&feature=related

Another version: start at 20 seconds (After Twinkle, Twinkle)
http://www.youtube.com/watch?v=b1q1pPI79TY&feature=related

Another version: Follow the Weasel (with equation)
http://www.youtube.com/watch?v=2lbABbfU6Zc

Another song: (Done on guitar with equation) 3:45 seconds...complete with intro
Song starts at 0:45 seconds
https://www.youtube.com/watch?v=9WbbyAq5BjE

Attachments

PopGoestheWeasel.mid