Before we begin, are there any questions from last day's work?

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) calculate the arc length of circle.
b) calculate the area of a sector of a circle.

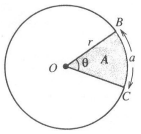
Project Renovation assignment past due.

Cake Decorating assignment past due.

6.9.1: Arc Length and Sector Area

Date: _____

There are problems that often occur in industry that involve arcs and sectors of circles.
Consider the following diagram:



A circle of radius r is drawn, with sector *BOC* bounded by 2 radii, *OB* and *OC*, and an arc *BC*, of length *a*.
The area of the sector is A,
and the sector angle at the centre *O* is θ, measured in degrees.

We can use the proportional relationship: $\frac{arc\ length}{circumference} = \frac{sector\ area}{area\ of\ circle} = \frac{sector\ angle}{complete\ rotation}$

So, if $\frac{a}{2\pi r} = \frac{A}{\pi r^2} = \frac{\theta}{360^\circ}$

$a = \frac{90}{360}(2\pi r)$

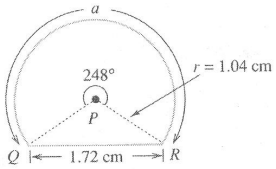
then $\frac{a}{2\pi r} = \frac{\theta}{360^\circ}$ and $\frac{A}{\pi r^2} = \frac{\theta}{360^\circ}$

and isolating, [*arc length*] $a = 2\pi r\left(\frac{\theta}{360^\circ}\right)$ and [*sector area*] $A = \pi r^2\left(\frac{\theta}{360^\circ}\right)$

Thus both the arc length, *a*, and sector area, *A*, can be calculated
once the radius, *r*, and the sector angle, θ, in degrees, are known.

Ex. 1 A cam for a sewing machine's stitching-control cycle is circular in shape, with a flat side, and has the dimensions shown.

a) Calculate the total perimeter of the cam. (to 3 decimal places)



$P = \text{arc} + \text{flat}$

$= \left(\frac{248}{360}\right)2\pi r + 1.72$

$= \frac{248}{360}(2\pi(1.04)) + 1.72$

$\doteq 4.5015 + 1.72$

$\doteq 4.502 + 1.72$

$\doteq 6.222$ cm

b) If the cam is 0.36 cm thick and is made from an alloy whose density is 3.8g/cm^3, determine the mass of the cam. (to 3 decimal places)

Hint: the cam is a prism, Volume $_{prism}$ = Area $_{base}$ x height, then mass = volume x density

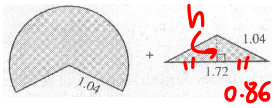
$A_{Sector} = \left(\frac{\theta}{360}\right)\pi r^2$

$= \frac{248}{360}(\pi(1.04)^2)$

$\doteq 2.3405$

$\doteq 2.341$

$A_{TRI} = \frac{1}{2}bh$



$\doteq \frac{1}{2}(1.72)(0.585)$

$\doteq 0.5031$

$\doteq 0.503$

$h^2 = 1.04^2 - 0.86^2$

$\doteq 0.342$

$h \doteq \sqrt{0.342}$

$\doteq 0.5848$

$\doteq 0.585$

$A_{cam} = A_{Sector} + A_{TRI}$

$\doteq 2.341 + 0.503$

$\doteq 2.844$ cm^2

$V_{cam} = A_{base} \times h$

$\doteq 2.844 \times 0.36$

$\doteq 1.0238$

$\doteq 1.024$ cm^3

Mass = Density x Volume

$\doteq 3.8\ \frac{g}{cm^3} \times 1.024\ cm^3$

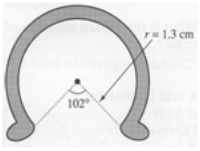
$\doteq 3.8912$

∴ the mass of the cam is 3.89 g.

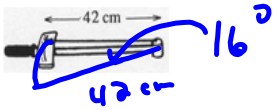
6.9.2 Arc Length and Sector Area

Date: _____

1. A snap-ring retainer clip with dimensions shown is part of a universal joint assembly, and fits snugly around the bearing cap when assembled. What length of the clip is in contact with the bearing cap? (to 2 decimal places)



2. The pointer on a torque wrench is 42 cm long, and moves through an angle of 16°. Through what distance does the tip of the pointer move? (to 1 decimal place)



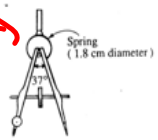
arc length $= \frac{\theta}{360}(2\pi r)$

$= \frac{16}{360}(2\pi(42))$

$\doteq 11.72$

$\doteq 11.7$ cm

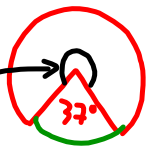
3. Calculate the length of the spring on the bow compasses shown. (to 2 decimal places)



$d = 1.8$ cm

$\therefore r = 0.9$ cm

$\therefore$ arc length



$360° - 37°$

$= 323°$

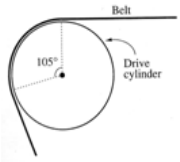
$= \left(\frac{\theta}{360}\right) 2\pi r$

$= \frac{323}{360}(2\pi(0.9))$

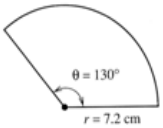
$\doteq 5.073$

$\doteq 5.07$ cm

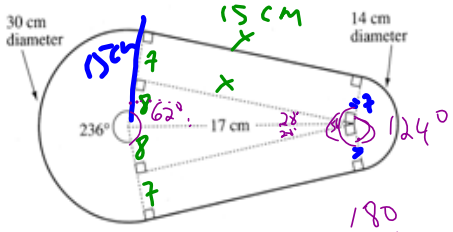
4. The belt on a copier machine is in contact with a drive cylinder over 105° of its surface, as shown. If the length of contact is 5.62 cm, what is the radius of the drive cylinder? (to 2 decimal places)



5. The vent cover on a forced air heating system is in the form of a sector of a circle. Determine the area of sheet metal used to make the vent cover. (to 1 decimal place)



6. The safety shield for a motor pulley drive has the dimensions shown.
   a) Calculate the perimeter of the shield. (to 1 decimal place)
   b) Determine the area of the shield. (to 1 decimal place)

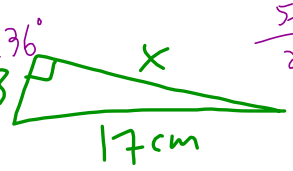


a) $P = (\quad + \quad) +$

$= \frac{236}{360}(2\pi(15)) + 15 + \frac{124}{360}(2\pi(7)) + 15$

$= 360° - 236°$

$= 124°$



180 / 56 / 236

$x^2 = 17^2 - 8^2$

$= 225$

$\therefore x = 15$ cm

Answers

1. 5.85 cm
2. 11.7 cm
3. 5.07 cm
4. 3.07 cm
5. 58.8 cm^2
6. (*a*) 106.9 cm
   (*b*) 846.4 cm^2

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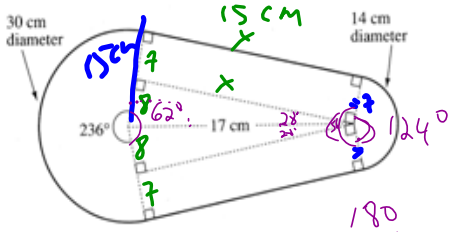
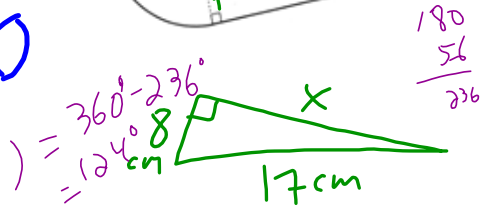


b) $A_{Total} = \bigcirc + 2Rect + 2\Delta s + \bigcirc$

$= \frac{236}{360}\pi(15)^2 + 2(15)(7)$

$+ 2\left(\frac{1}{2}(8)(15)\right)$

$+ \frac{124}{360}\pi(7)^2$



180
56
236

$x^2 = 17^2 - 8^2$

$= 225$

$\therefore x = 15$ cm

Answers

1. 5.85 cm
2. 11.7 cm
3. 5.07 cm
4. 3.07 cm
5. 58.8 cm^2
6. (*a*) 106.9 cm
   (*b*) 846.4 cm^2