**Before we begin, are there any questions from last day's work?**

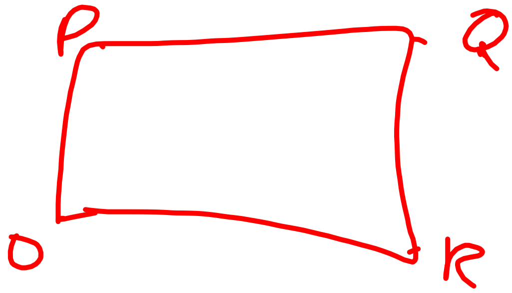
# Today's Learning Goal(s):

7, 8, 17

By the end of the class, I will be able to:

a) find the equation of a circle that has a radius "***r***", with centre (0, 0).
b) use key points to sketch a circle.

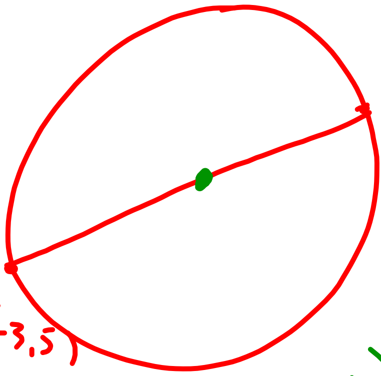
p.89 #7




$m_{PQ} =$ $m_{OR} =$

$m_{OP} =$ $m_{QR} =$

p.89 #8




a) $C = M_{MN}$

$= \left(\frac{9+(-3)}{2}, \frac{7+5}{2}\right)$

$= (3, 6)$

b) $r = |CN|$

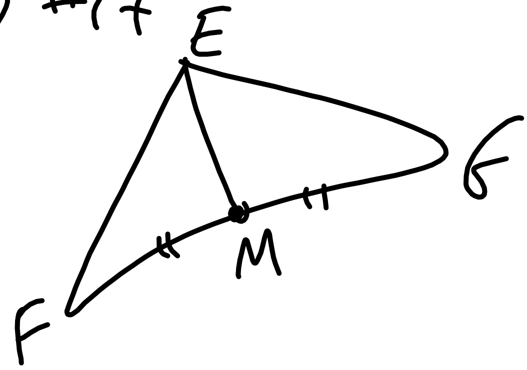
$= \sqrt{(9-3)^2 + (7-6)^2}$

$= \sqrt{6^2 + 1^2}$

$= \sqrt{37}$ units

p.90 #17

a)




① Midpoint of FG

② Slope of EM

③ eq'n to find "b"

④ conclusion.

b) $|EM|$

MPM 2DI Warm-up  Date: Oct. 4/16

1. Evaluate. (No decimals allowed!)

a) $\left(\sqrt{9}\right)^2$ b) $\sqrt{9^2}$ c) $\left(\sqrt{16}\right)^2$ d) $\sqrt{11^2}$ e) $\left(\sqrt{3}\right)^2$ f) $\left(\sqrt{5}\right)^2$ g) $\left(-\sqrt{5}\right)^2$

a) $=(3)^2$
$=9$

b) $=\sqrt{81}$
$=9$

c) $=4^2$
$=16$

d) $=\sqrt{121}$
$=11$

e) $=3$

f) $=5$

g) $=(-\sqrt{5})(-\sqrt{5})$
$=+\sqrt{25}$
$=5$

2. Determine the distance of each point from the origin, O ( 0 , 0 ).

*Recall:* $|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $(x_1, y_1)$

$x_2$ $y_2$

a) $A(0,10)$ b) $B(6,8)$ c) $C(-8,6)$ d) $D\left(\sqrt{19}, -9\right)$

a) $|OA|$
$=\sqrt{(0-0)^2+(10-0)^2}$
$=\sqrt{0^2+10^2}$
$=\sqrt{100}$
$=10$ units

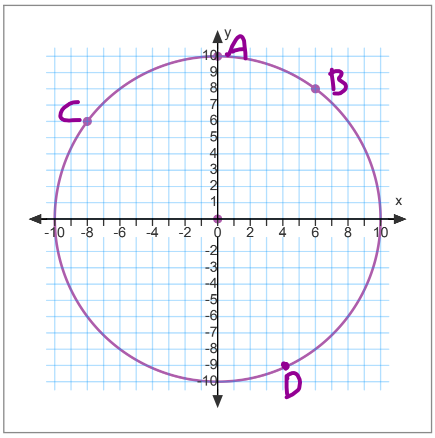
b) $|OB|$
$=\sqrt{(6-0)^2+(8-0)^2}$
$=\sqrt{6^2+8^2}$
$=\sqrt{36+64}$
$=\sqrt{100}$
$=10$ units

c) $|OC|$
$=\sqrt{(-8-0)^2+(6-0)^2}$
$=\sqrt{(-8)^2+6^2}$
$=\sqrt{64+36}$
$=\sqrt{100}$
$=10$ units

d) $|OD|$
$=\sqrt{(\sqrt{19}-0)^2+(-9-0)^2}$
$=\sqrt{(\sqrt{19})^2+(-9)^2}$
$=\sqrt{19+81}$
$=\sqrt{100}$
$=10$ units

1

How can one describe, *using an equation*, ***any*** point that is **10** units in length from the origin?




We have $r = 10$, $P_1$ is the origin: **O(0, 0)**, and $P_2$ is any other point: **P($x$, $y$)**.

$$|OP| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$10 = \sqrt{(x-0)^2 + (y-0)^2}$$
$$10 = \sqrt{x^2 + y^2}$$
$$(10)^2 = \left(\sqrt{x^2 + y^2}\right)^2$$
$$100 = x^2 + y^2$$

How can one describe, *using an equation*, ***any*** point that is $r$ units in length from the origin?

$$|OP| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$r = \sqrt{(x-0)^2 + (y-0)^2}$$
$$r = \sqrt{x^2 + y^2}$$
$$r^2 = \left(\sqrt{x^2 + y^2}\right)^2$$
$$r^2 = x^2 + y^2$$
$$x^2 + y^2 = r^2$$

MPM 2DI 2.4 Equation for a Circle Date: Oct. 4/16

## Circle

A **circle** is the set of all points in the plane that are equidistant from a fixed point called the centre. This distance is called the radius.

In general, the equation of a circle with centre ( 0 , 0 ) and radius, r, is: $x^2 + y^2 = r^2$

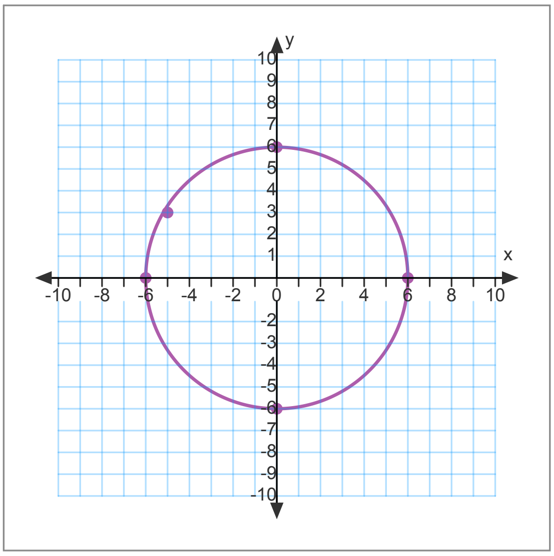
Ex. 1 Given circle: $x^2 + y^2 = 36$

i) Determine:
   a) the radius
   b) the diameter
   c) the coordinates of 4 points on the circle
ii) sketch the graph
iii) Is the point ( -5 , 3 ) on the circle?

$r^2 = 36$
$r = \sqrt{36}$
$= 6$ units

$d = 2r$
$= 2(6)$
$= 12$ units

$(0, 6)$ $(6, 0)$ $(-6, 0)$ $(0, -6)$




Check the equation

$LS = x^2 + y^2$ $RS = 36$
$= (-5)^2 + (3)^2$
$= 25 + 9$
$= 34$

$\therefore LS \neq RS$
$\therefore (-5, 3)$ is NOT on the circle $x^2 + y^2 = 36$

$\therefore LS < RS$
pt $(-5, 3)$ is INSIDE the circle.

Ex. 2 a) Find the equation of the circle, centred at the origin, through the point P( 6 , -3 ).
b) What is the diameter of this circle.

a) $x^2 + y^2 = r^2$

$(6)^2 + (-3)^2 = r^2$

$36 + 9 = r^2$

$45 = r^2$

$\therefore x^2 + y^2 = 45$ is the equation.

b) $d = 2r$

$= 2(3\sqrt{5})$

$= 6\sqrt{5}$ units

$r^2 = 45$

$r = \sqrt{45}$

$= \sqrt{9}\sqrt{5}$

$= 3\sqrt{5}$ units

$x^2 + y^2 = 45$

d=$6\sqrt{5}$units

Note: Given a point ( a , b ) and circle $x^2 + y^2 = r^2$ :

If $a^2 + b^2 = r^2 \rightarrow$ the point ( a , b ) is ________________ the circle.

If $a^2 + b^2 < r^2 \rightarrow$ the point ( a , b ) is ________________ the circle.

If $a^2 + b^2 > r^2 \rightarrow$ the point ( a , b ) is ________________ the circle.

Today's entertainment: pp. 96-98 # 1bcef, 2cd, 3ac, 4bdf, 6, 7, 8, 9, 11

***Solutions to 7, 8, 9, 11 follow.***

**Chapter 2 Section 4** **Question 7** **Page 97**

**a)** $x^2 + y^2 = 100$

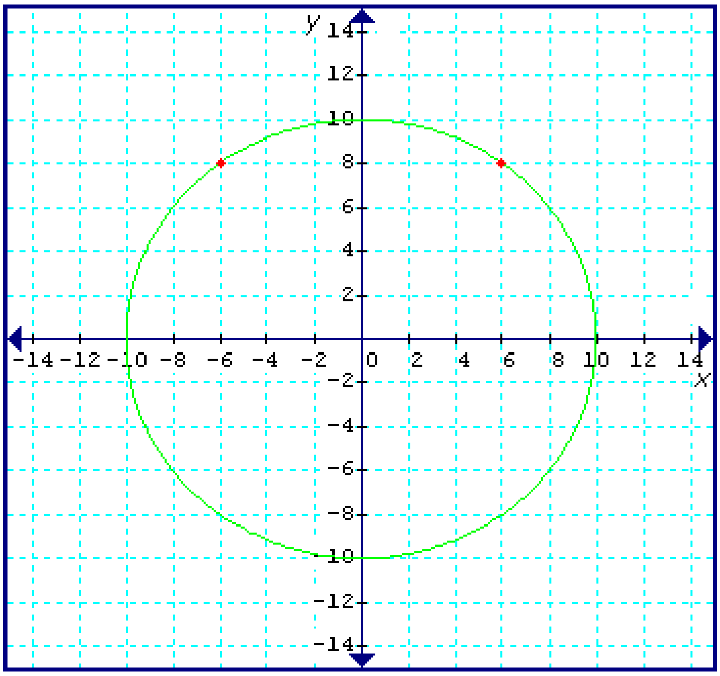
$a^2 + 8^2 = 100$

$a^2 = 36$

$a = \pm 6$

Substituting the coordinates into the equation gives $a^2 = 36$. Therefore, $a$ can be either 6 or $-6$.

**b)**



**Chapter 2 Section 4 Question 8 Page 97**

**a)** Substitute $r = 8$ into the formula for the circumference of a circle.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(8) \\ &\doteq 50.3 \end{aligned}$$

Approximately 50.3 m of fencing is required.

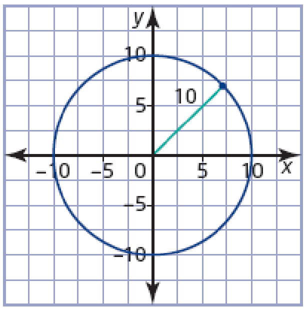
**b)** Substitute $r = 8$ into the area formula for a circle.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(8)^2 \\ &\doteq 201 \end{aligned}$$

The area of the corral is about 201 $m^2$.

**Chapter 2 Section 4** **Question 9** **Page 97**

**a)**




**b)** $r_1 = \sqrt{x^2 + y^2}$
$= \sqrt{(-8)^2 + 6^2}$
$= \sqrt{64 + 36}$
$= \sqrt{100}$
$= 10$

$r_1 = \sqrt{x^2 + y^2}$
$= \sqrt{6^2 + 8^2}$
$= \sqrt{36 + 64}$
$= \sqrt{100}$
$= 10$

The points P$(-8, 6)$ and Q$(6, 8)$ both lie on the circle.

**c)** Find the midpoint of chord PQ.

$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$= \left(\frac{-8+6}{2}, \frac{6+8}{2}\right)$
$= (-1, 7)$

Find the slope of chord PQ.

$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{8-6}{6+8}$
$= \frac{2}{14}$
$= \frac{1}{7}$

The slope of the right bisector of PQ must be $-7$.
Substitute $m = -7$ and the coordinates of the midpoint to find $b$.
$y = mx + b$
$7 = -7(-1) + b$
$0 = b$

The equation of the right bisector of PQ is $y = -7x$.

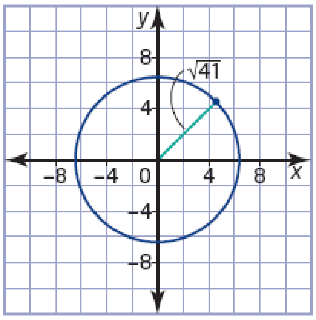
**d)** The coordinates $(0, 0)$ satisfy the equation $y = -7x$.

**e)** Answers may vary. For example:

Since the endpoints of any chord lie on a circle, they are equidistant from the centre of the circle. All points equidistant from the endpoints of a line segment lie on the right bisector of the line segment. Therefore, the right bisector of any chord of a circle passes through the centre of the circle.

**Chapter 2 Section 4** **Question 11** **Page 98**

**a)**




**b)** $r_1 = \sqrt{x^2 + y^2}$
$= \sqrt{(-4)^2 + 5^2}$
$= \sqrt{16 + 25}$
$= \sqrt{41}$

$r_1 = \sqrt{x^2 + y^2}$
$= \sqrt{(-5)^2 + (-4)^2}$
$= \sqrt{25 + 16}$
$= \sqrt{41}$

The points U(–4, 5) and V(–5, –4) both lie on the circle. UV is a chord.

**c)** Find the slope of chord UV.

$$m_{UV} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-4 - 5}{-5 - (-4)}$$
$$= \frac{-9}{-1}$$
$$= 9$$

The slope of a line perpendicular to the chord is $-\frac{1}{9}$.

Substitute $m = -\frac{1}{9}$ and the coordinates of the origin to find $b$.

$$y = mx + b$$
$$0 = -\frac{1}{9}(0) + b$$
$$0 = b$$

**d)** Find the coordinates of the midpoint of chord UV.

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-4 + (-5)}{2}, \frac{5 + (-4)}{2}\right)$$
$$= \left(-\frac{9}{2}, \frac{1}{2}\right)$$

Check that these coordinates satisfy the equation of the line.

**L.S.** $= y$
$= \frac{1}{2}$

**R.S.** $= -\frac{1}{9}x$
$= -\frac{1}{9}\left(-\frac{9}{2}\right)$
$= \frac{1}{2}$

**L.S. = R.S.**

The coordinates of the midpoint satisfy the equation. The line passes
chord.

The equation of the line from the origin perpendicular to the chord UV is $y = -\frac{1}{9}x$.