

## Today's Learning Goal(s):

By the end of the class, I will:

- a) know all formulas needed for this unit.
- b) have anchor charts created for all unit problems.
- c) be able to create a practice test to prepare for the unit summative.

(This means you must understand all the topics from this unit, and be able to provide a mathematical example for each. You may use the assigned homework as a guide.)

Need to know, from Unit 2: Analytic Geometry

Formulas:

slope	midpoint	length	equation of a circle
$m = \frac{y_2 - y_1}{x_2 - x_1}$	$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$x^2 + y^2 = r^2$

Anchor Charts

- finding the equation of the median of a triangle from a given vertex
- finding the equation of the altitude of a triangle from a given vertex
- finding the equation of the perpendicular bisector of a line segment
- finding the point of intersection of any of the above

See examples next page.

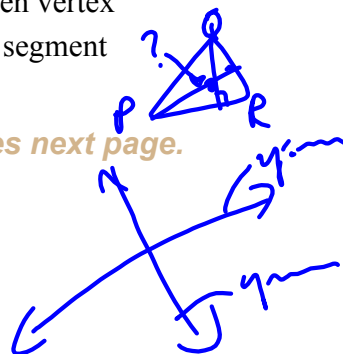
Other Concepts

- finding the equation of a line
- is a point on a line?
- finding the equation of a circle
- is a point inside, outside or on a circle?
- chords, diameters, and radius of a circle
- parallel vs. perpendicular lines and the relationship between their slopes
- converting an entire radical to a mixed radical in lowest terms
- proving a geometric figure is:

a right triangle

an equilateral triangle  
an isosceles triangle  
a scalene triangle

a square  
a rectangle

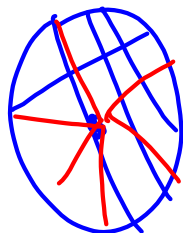


$< r^2$

$$d = 7$$

$$x^2 + y^2 = (3.5)^2$$

$$= 12.25$$



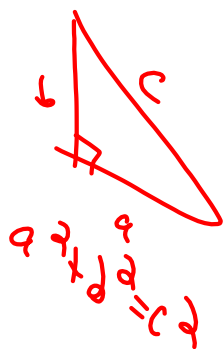
$$m_1 = m_2$$

parallel  
↳ slopes are equal

$$\sqrt{50}$$

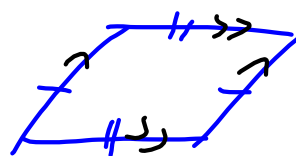
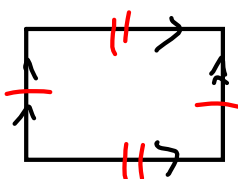
$$= \sqrt{25} \sqrt{2}$$

$$= 5\sqrt{2}$$



perpendicular  
↳ slopes are negative reciprocals

$$m_3 = \frac{-1}{m_4}$$



$$(7, 2) \quad r = ?$$

$$x^2 + y^2 = r^2$$

$$(7)^2 + (2)^2 = r^2$$

$$49 + 4 = r^2$$

$$53 = r^2$$

$$\therefore x^2 + y^2 = 53 \text{ is the equation.}$$

Suppose you are given only the coordinates for a triangle, CDE.

*Using words only, **explain*** how you would find the equation of the **altitude** from vertex **E**.

**OR**

Suppose you are given only the coordinates for a triangle, GHI

*Using words only, **explain*** how you would find the equation of the **median** from vertex **H**.

**OR**



A triangle has vertices  $P( \quad , \quad )$ ,  $Q( \quad , \quad )$  and  $R( \quad , \quad )$ .

Find an equation for the **median** from vertex **P**.

**Note: Extra review, with answers is posted on the website.**

Today's Review practice:

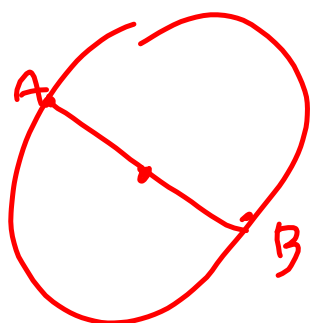
pp. 100-103 #2b, 4, 8, 11a, 12abc\*,  
14ab, 15, 18

Error in final answer for 12c: it should read "... $y = x - 5$ ..."

*Today's solutions follow on the remaining slides.*

Circle Hwk: 8, 6, 7, 9

p.97 #6 A(-4,3) B(4,-3)



$$x^2 + y^2 = r^2$$

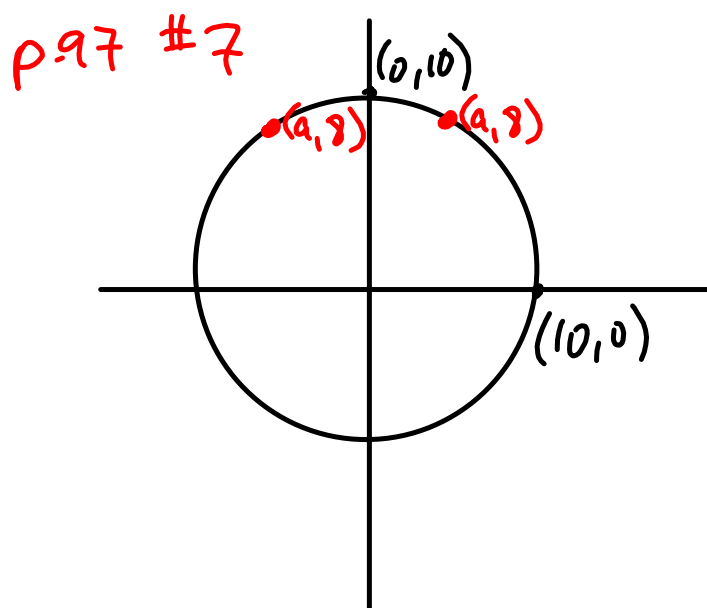
(use A)

$$(-4)^2 + (3)^2 = r^2$$

$$16 + 9 = r^2$$

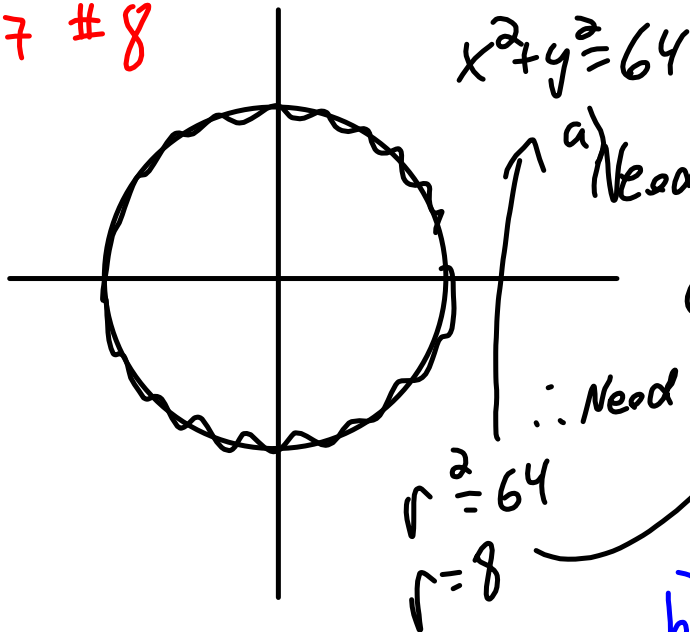
$$25 = r^2$$

$\therefore x^2 + y^2 = 25$  is the equation.



$$\begin{aligned}x^2 + y^2 &= 100 \\a^2 + 8^2 &= 100 \\a^2 + 64 &= 100 \\a^2 &= 100 - 64 \\&= 36 \\a &= \pm\sqrt{36} \\&= \pm 6 \\\therefore (6, 8) \text{ or } (-6, 8)\end{aligned}$$

part #8



$$x^2 + y^2 = 64$$

a) Need circumference

$$C = 2\pi r$$

∴ Need r. →  $C = 2\pi(8)$ 

$$= 16\pi$$

$$\approx 50.24 \text{ units}$$

$$r^2 = 64$$

$$r = 8$$

b) A = ?

$$A = \pi r^2$$

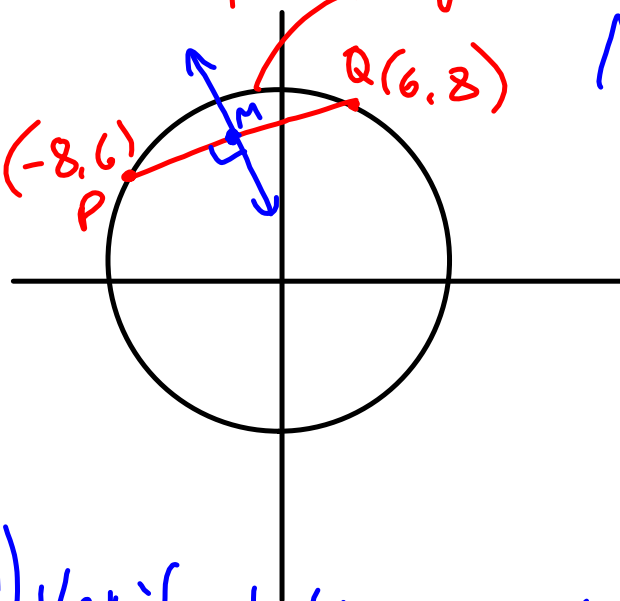
$$= \pi(8)^2$$

$$= 64\pi$$

$$= 201.06 \text{ units}^2$$



$P \neq Q \neq 9$   $x^2 + y^2 = 100$



$$M_{PQ} \left( \frac{-8+6}{2}, \frac{6+8}{2} \right)$$

$$M \left( \frac{-2}{2}, \frac{14}{2} \right)$$

$$M(-1, 7)$$

$$m_{PQ} = \frac{8-6}{6-(-8)}$$

$$= \frac{2}{14}$$

$$= \frac{1}{7}$$

$$\begin{aligned} & \rightarrow M_{\perp} = -7 \\ & \therefore y = -7x + b \\ & (7) = -7(-1) + b \\ & 7 = 7 + b \\ & 0 = b \end{aligned}$$

d) Verify  $\perp$  through  $C(0,0)$   
 $LS = y$   $RS = -7x$

$y = -7x$  is the  
 perp of the right  
 bisector of  $PQ$ .

p. 100 #2b

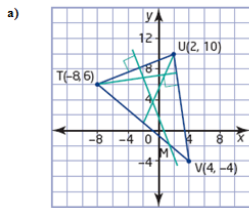
$$\begin{aligned}\mathbf{b)} \quad (x, y) &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{4 + 4}{2}, \frac{8 + (-2)}{2} \right) \\ &= (4, 3)\end{aligned}$$

The coordinates of the midpoint of  $\underline{LN}$ (4, 8) and N(4, -2) are (4, 3).

## p. 100 #4

Chapter 2 Review

Question 4 Page 100



b) Find the coordinates of the midpoint of TV.

$$\begin{aligned}(x, y) &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{-8 + 4}{2}, \frac{6 + (-4)}{2} \right) \\ &= (-2, 1)\end{aligned}$$

Find the slope of the median.

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{10 - 1}{2 - (-2)} \\ &= \frac{9}{4}\end{aligned}$$

Substitute  $m = \frac{9}{4}$  and the coordinates of one endpoint, say  $(-2, 1)$ , to find  $b$ .

$$\begin{aligned}y &= mx + b \\ 1 &= \frac{9}{4}(-2) + b \\ 1 &= -\frac{9}{2} + b \\ \frac{11}{2} &= b\end{aligned}$$

The equation of the median from vertex U is  $y = \frac{9}{4}x + \frac{11}{2}$ .

d) Find the coordinates of the midpoint of TU.

$$\begin{aligned}(x, y) &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{-8 + 2}{2}, \frac{6 + 10}{2} \right) \\ &= (-3, 8)\end{aligned}$$

Find the slope of TU.

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{10 - 6}{2 - (-8)} \\ &= \frac{4}{10} \\ &= \frac{2}{5}\end{aligned}$$

The slope of TU is  $\frac{2}{5}$ . The slope of the right bisector of TU is  $-\frac{5}{2}$ .Substitute  $m = -\frac{5}{2}$  and the coordinates of the midpoint to find  $b$ .

$$\begin{aligned}y &= mx + b \\ 8 &= -\frac{5}{2}(-3) + b \\ 8 &= \frac{15}{2} + b \\ \frac{1}{2} &= b\end{aligned}$$

The equation of the right bisector of TU is  $y = -\frac{5}{2}x + \frac{1}{2}$ .

c) Find the slope of UV.

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - 10}{4 - 2} \\ &= -7\end{aligned}$$

The slope of side UV is  $-7$ . The slope of the altitude from vertex T is  $\frac{1}{7}$ .Substitute  $m = \frac{1}{7}$  and the coordinates of point T to find  $b$ .

$$\begin{aligned}y &= mx + b \\ 6 &= \frac{1}{7}(-8) + b \\ 6 &= -\frac{8}{7} + b \\ \frac{50}{7} &= b\end{aligned}$$

The equation of the altitude from vertex T is  $y = \frac{1}{7}x + \frac{50}{7}$ .

## p. 101 #8

Chapter 2 Review

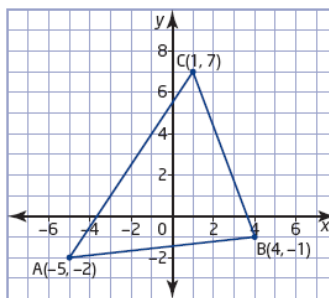
Question 8 Page 101

a) Find the coordinates of the midpoint of side BC.

$$\begin{aligned}(x, y) &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{1 + 4}{2}, \frac{7 + (-1)}{2} \right) \\ &= \left( \frac{5}{2}, 3 \right)\end{aligned}$$

Find the length of the median.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left( \frac{5}{2} - (-5) \right)^2 + (3 - (-2))^2} \\ &= \sqrt{\left( \frac{15}{2} \right)^2 + 5^2} \\ &= \sqrt{\frac{325}{4}}\end{aligned}$$

The length of the median from vertex A is  $\sqrt{\frac{325}{4}}$ .

b) Find the lengths of the sides.

$$\begin{aligned}AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - (-5))^2 + (-1 - (-2))^2} \\ &= \sqrt{9^2 + 1^2} \\ &= \sqrt{82}\end{aligned}$$

$$\begin{aligned}CB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 1)^2 + (-1 - 7)^2} \\ &= \sqrt{3^2 + (-8)^2} \\ &= \sqrt{73}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1 - (-5))^2 + (7 - (-2))^2} \\ &= \sqrt{6^2 + 9^2} \\ &= \sqrt{117}\end{aligned}$$

The perimeter of the triangle is  $\sqrt{82} + \sqrt{73} + \sqrt{117}$ , or about 28.4.

p. 102 #11a

**Chapter 2 Review****Question 11 Page 102**

a) Find the slopes of the sides of  $\triangle DEF$ .

$$\begin{aligned} m_{DE} &= \frac{y_2 - y_1}{x_2 - x_1} & m_{EF} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 7}{-4 - (-2)} & &= \frac{-2 - 2}{6 - (-4)} \\ &= \frac{-5}{-2} & &= \frac{-4}{10} \\ &= \frac{5}{2} & &= -\frac{2}{5} \end{aligned}$$

The slopes are negative reciprocals. DE is perpendicular to EF. Therefore,  $\triangle DEF$  is a right triangle with  $\angle DEF = 90^\circ$ .

p. 102 #12abc

## Chapter 2 Review

## Question 12 Page 102

$$\begin{aligned}
 \text{a) } d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(65 - 45)^2 + (40 - 60)^2} \\
 &= \sqrt{20^2 + (-20)^2} \\
 &= \sqrt{800} \\
 &\approx 28.3
 \end{aligned}$$

The distance from A to B is about 28.3 km.

$$\begin{aligned}
 \text{b) } (x, y) &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left( \frac{45 + 65}{2}, \frac{60 + 40}{2} \right) \\
 &= (55, 50)
 \end{aligned}$$

c) Find the equation of line representing the branch pipeline.

$$\begin{aligned}
 m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{40 - 60}{65 - 45} \\
 &= \frac{-20}{20} \\
 &= -1
 \end{aligned}$$

The slope of the line representing the pipeline is  $-1$ . The slope of the branch pipeline is  $1$ . Substitute  $m = 1$  and the coordinates of the midpoint to find  $b$ .

$$y = mx + b$$

$$50 = 1(55) + b$$

$$-5 = b$$

The equation of the line representing the branch line is  $y = x - 5$ . Check if the given point satisfies the equation.

$$\begin{array}{ll}
 \text{L.S.} = y & \text{R.S.} = x - 5 \\
 = 54 & = 63 - 5 \\
 & = 58
 \end{array}$$

$$\underline{\text{L.S.} \neq \text{R.S.}}$$

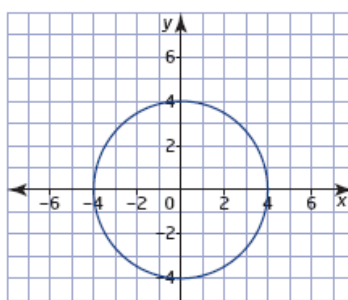
Point C is not on the branch pipeline.

p. 102 #14ab

Chapter 2 Review

Question 14 Page 102

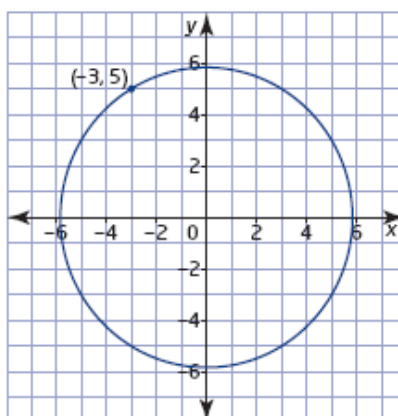
a)  $x^2 + y^2 = 16$



b)  $x^2 + y^2 = r^2$

$(-3)^2 + 5^2 = r^2$

$34 = r^2$

An equation for the circle is  $x^2 + y^2 = 34$ .

p. 103 #15

**a)**  $x^2 + y^2 = r^2$

$$x^2 + y^2 = \left(4\frac{1}{2}\right)^2$$

$$x^2 + y^2 = \frac{81}{4}$$

**b)**  $x^2 + y^2 = r^2$

$$x^2 + y^2 = \left(\frac{14}{2}\right)^2$$

$$x^2 + y^2 = 49$$

**c)**  $x^2 + y^2 = r^2$

$$x^2 + y^2 = \left(\sqrt{12}\right)^2$$

$$x^2 + y^2 = 12$$

**d)**  $x^2 + y^2 = r^2$

$$4^2 + 7^2 = r^2$$

$$65 = r^2$$

An equation for the circle is  $x^2 + y^2 = 65$ .



p. 103 #18

**Chapter 2 Review                      Question 18   Page 103**

The radius of the signal area is 20 km. Check that the cell phone user is within the circular area.

$$\begin{aligned}r_1 &= \sqrt{x^2 + y^2} \\ &= \sqrt{13^2 + 15^2} \\ &= \sqrt{169 + 225} \\ &= \sqrt{394} \\ &\doteq 19.8\end{aligned}$$

The user is less than 20 km from the tower. Signals can be received.

*Tomorrow's solutions from p.104*

**Chapter 2 Chapter Test      Question 5    Page 104**

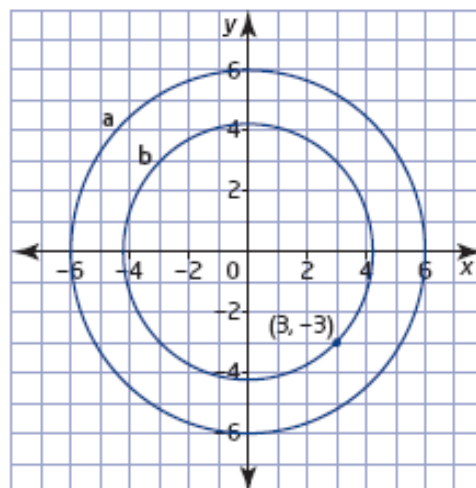
a)  $x^2 + y^2 = 36$

b)  $x^2 + y^2 = r^2$

$$3^2 + (-3)^2 = r^2$$

$$18 = r^2$$

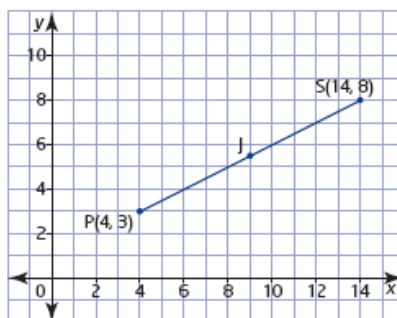
An equation for the circle is  $x^2 + y^2 = 18$ .



## Chapter 2 Chapter Test Question 7 Page 104

$$\begin{aligned}
 \text{a) } PS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(14 - 4)^2 + (8 - 3)^2} \\
 &= \sqrt{10^2 + 5^2} \\
 &= \sqrt{125} \\
 &\approx 11.2
 \end{aligned}$$

The schools are about 11.2 km apart.



$$\begin{aligned}
 \text{b) } (x, y) &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left( \frac{4 + 14}{2}, \frac{3 + 8}{2} \right) \\
 &= (9, 5.5)
 \end{aligned}$$

The coordinates of Jason's home are  $(9, 5.5)$ .

c) Answers may vary. For example:

Any point on the perpendicular bisector of PS will be equidistant from the two schools.

d) Find the slope of PS.

$$\begin{aligned}
 m_{PS} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{8 - 3}{14 - 4} \\
 &= \frac{5}{10} \\
 &= \frac{1}{2}
 \end{aligned}$$

The slope of the right bisector of PS is  $-2$ . Substitute  $m = -2$  and the coordinates of the midpoint to find  $b$ .

$$y = mx + b$$

$$5.5 = -2(9) + b$$

$$5.5 = -18 + b$$

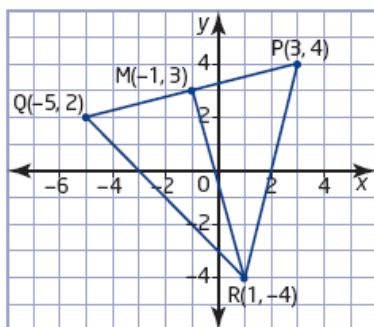
$$23.5 = b$$

$$y = -2x + 23.5$$

The equation of the right bisector of PS is  $y = -2x + 23.5$ .

## Chapter 2 Chapter Test Question 9 Page 104

a)



b) Find the coordinates of the midpoint, M, of QP.

$$\begin{aligned}(x, y) &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{-5 + 3}{2}, \frac{2 + 4}{2} \right) \\ &= (-1, 3)\end{aligned}$$

Find the slope of the median.

$$\begin{aligned}m_{RM} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - 3}{1 - (-1)} \\ &= \frac{-7}{2} \\ &= -\frac{7}{2}\end{aligned}$$

Substitute  $m = -\frac{7}{2}$  and the coordinates of one endpoint, say  $(-1, 3)$ , to find  $b$ .

$$\begin{aligned}y &= mx + b \\ 3 &= -\frac{7}{2}(-1) + b \\ 3 &= \frac{7}{2} + b \\ -\frac{1}{2} &= b\end{aligned}$$

The equation of the median from vertex R is  $y = -\frac{7}{2}x - \frac{1}{2}$ .

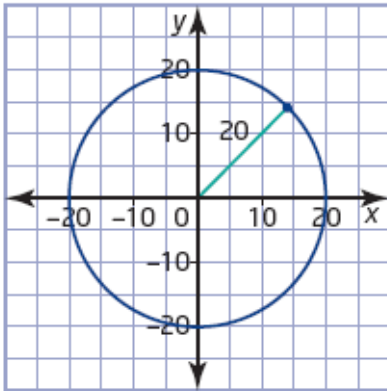
c) Find the slope of side QP.

$$\begin{aligned}m_{QP} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 2}{3 - (-5)} \\ &= \frac{2}{8} \\ &= \frac{1}{4}\end{aligned}$$

The slope of QP is not the negative reciprocal of the slope of RM. The median is not an altitude.

## Chapter 2 Chapter Test Question 13 Page 105

a)



b) The boundary of the area is described by the equation  $x^2 + y^2 = 400$ .

c)

$$\begin{aligned} r_1 &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-8)^2 + 16^2} \\ &= \sqrt{64 + 256} \\ &= \sqrt{320} \\ &\approx 17.9 \end{aligned}$$

$$\begin{aligned} r_1 &= \sqrt{x^2 + y^2} \\ &= \sqrt{4^2 + 20^2} \\ &= \sqrt{16 + 400} \\ &= \sqrt{416} \\ &\approx 20.4 \end{aligned}$$

Since  $r_1 < 20$ , Arif is inside the circle.

Since  $r_1 > 20$ , Diane is outside the circle.

Arif is in range, but Diane is not.

$$\begin{aligned} \text{d) } d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-8 - 4)^2 + (16 - 20)^2} \\ &= \sqrt{(-12)^2 + (-4)^2} \\ &= \sqrt{160} \\ &\approx 12.6 \end{aligned}$$

Diane and Arif are about 12.6 km apart. They are within range of each other.