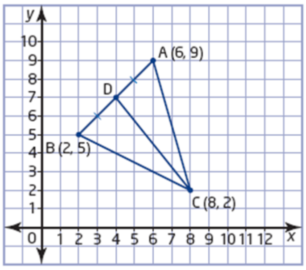
Readiness Check

1. Suppose you are given only the coordinates for a triangle, TUV.

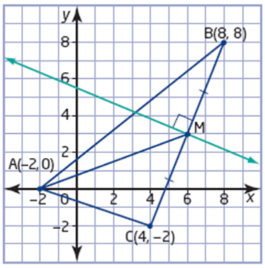
   Using words only, **and without looking at your notes**:

   a) **explain** how you would find the equation of the **median** from vertex **T**.
   b) **explain** how you would find the equation of the **right bisector** of **TU**.
   c) **explain** how you would find the equation of the **altitude** from vertex **U**.

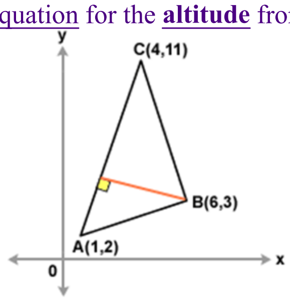
2. Find an equation for the **median** from vertex **C**.



3. Find an equation for the **right bisector** of **BC**.



4. Find an equation for the **altitude** from vertex **B**.



$y=\frac{-5}{4}x+12$ $y=\frac{-2}{5}x+\frac{27}{5}$ $y=\frac{-1}{3}x+5$

2. Determine an equation for the right bisector of the line segment with endpoints C(-5 , -3) and D(4 , -2).

Today's Learning Goal(s):

By the end of the class, I will:

a) know all formulas needed for this unit.
b) have anchor charts created for all unit problems.
c) be able to create a practice test to prepare for the unit summative.
(This means you must understand all the topics from this unit, and be able to provide a mathematical example for each. You may use the assigned homework as a guide.)

Need to know, from Unit 2: Analytic Geometry

Formulas:

slope	midpoint	length	equation of a circle
$m = \frac{y_2 - y_1}{x_2 - x_1}$	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$x^2 + y^2 = r^2$

Anchor Charts

finding the equation of the median of a triangle from a given vertex

finding the equation of the altitude of a triangle from a given vertex

finding the equation of the perpendicular bisector of a line segment

finding the point of intersection of any of the above

Other Concepts

finding the equation of a line

is a point on a line?

finding the equation of a circle

is a point inside, outside or on a circle?

chords, diameters, and radius of a circle

parallel vs. perpendicular lines and the relationship between their slopes

converting an entire radical to a mixed radical in lowest terms

proving a geometric figure is:

a right triangle

an equilateral triangle
an isosceles triangle
a scalene triangle

a square
a rectangle

Note: Extra review is posted on the website.

Yesterday's Review practice:

pp. 100-103 #2b, 4, 8, 11a, 12abc*,

14ab, 15, 18

Error in final answer for 12c: it should read "…y = x – 5…"

Today's Review practice:

p. 104 #5, 7, 9, 13

Yesterday's solutions follow.

Today's solutions follow those.

Use previous answers if parts a) b) c) etc.

Remind students "exact" means no decimals!

$= \sqrt{2}$

$\doteq 1.159$ cm

$\sqrt{40}$

$= \sqrt{4}\sqrt{10}$

$= 2\sqrt{10}$ units

p. 100 #2b

b) $$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{4 + 4}{2}, \frac{8 + (-2)}{2}\right)$$
$$= (4, 3)$$

The coordinates of the midpoint of L(4, 8) and N(4, –2) are (4, 3).

p. 100 #4

b) Find the coordinates of the midpoint of TV.

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-8 + 4}{2}, \frac{6 + (-4)}{2}\right)$$

$$M = (-2, 1)$$

Find the slope of the median.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{10 - 1}{2 - (-2)}$$

$$= \frac{9}{4}$$

Substitute $m = \frac{9}{4}$ and the coordinates of one endpoint, say (−2, 1), to find b.

$$y = mx + b$$

$$1 = \frac{9}{4}(\cancel{-2})+b \quad -4$$

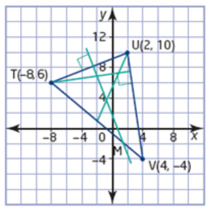
$$\frac{2}{2} = -\frac{9}{2} + b$$

$$\frac{11}{2} = b$$

The equation of the median from vertex U is $y = \frac{9}{4}x + \frac{11}{2}$.

Chapter 2 Review **Question 4** **Page 100**

a)



c) Find the slope of UV.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-4 - 10}{4 - 2}$$

$$= -7$$

$$y = \frac{1}{7}x + b$$

The slope of side UV is −7. The slope of the altitude from vertex T is $\frac{1}{7}$.

Substitute $m = \frac{1}{7}$ and the coordinates of point T to find b.

$$y = mx + b$$

$$6 = \frac{1}{7}(-8) + b$$

$$6 = -\frac{8}{7} + b \rightarrow 6 + \frac{8}{7} = b$$

$$\frac{42}{7} + \frac{8}{7} = b$$

$$\frac{50}{7} = b$$

The equation of the altitude from vertex T is $y = \frac{1}{7}x + \frac{50}{7}$.

d) Find the coordinates of the midpoint of TU.

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-8 + 2}{2}, \frac{6 + 10}{2}\right)$$

$$= (-3, 8)$$

Find the slope of TU.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{10 - 6}{2 - (-8)}$$

$$= \frac{4}{10}$$

$$= \frac{2}{5}$$

The slope of TU is $\frac{2}{5}$. The slope of the right bisector of TU is $-\frac{5}{2}$.

Substitute $m = \frac{2}{5}$ and the coordinates of the midpoint to find b.

$$y = mx + b$$

$$8 = -\frac{5}{2}(-3) + b$$

$$8 = \frac{15}{2} + b$$

$$\frac{1}{2} = b$$

$$8 - \frac{15}{2} = b$$

$$\frac{16}{2} - \frac{15}{2} = b$$

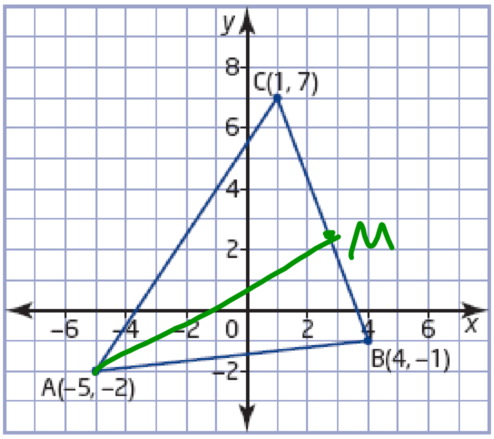
The equation of the right bisector of TU is $y = -\frac{5}{2}x + \frac{1}{2}$.

p. 101 #8 ✓

Chapter 2 Review **Question 8** **Page 101**

a) Find the coordinates of the midpoint of side BC.

$$(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{1+4}{2}, \frac{7+(-1)}{2}\right)$$
$$= \left(\frac{5}{2}, 3\right)$$



|AM|

Find the length of the median.

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{\left(\frac{5}{2}-(-5)\right)^2+(3-(-2))^2}$$
$$= \sqrt{\left(\frac{15}{2}\right)^2+5^2}$$
$$= \sqrt{\frac{325}{4}}$$

$$\sqrt{\left(\frac{5}{2}+\frac{10}{2}\right)^2+(3+2)^2}$$

$$\sqrt{\frac{15^2}{2^2}+5^2}$$
$$= \sqrt{\frac{225}{4}+25}$$
$$= \sqrt{\frac{225}{4}+\frac{100}{4}}$$
$$= \sqrt{\frac{325}{4}}$$
$$= \frac{\sqrt{25}\sqrt{13}}{\sqrt{4}}$$
$$= \frac{5\sqrt{13}}{2} \text{ units}$$

The length of the median from vertex A is $\sqrt{\frac{325}{4}}$.

b) Find the lengths of the sides.

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(4-(-5))^2+(-1-(-2))^2}$$
$$= \sqrt{9^2+1^2}$$
$$= \sqrt{82}$$

$$CB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(4-1)^2+(-1-7)^2}$$
$$= \sqrt{3^2+(-8)^2}$$
$$= \sqrt{73}$$

$$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(1-(-5))^2+(7-(-2))^2}$$
$$= \sqrt{6^2+9^2}$$
$$= \sqrt{117}$$

The perimeter of the triangle is $\sqrt{82}+\sqrt{73}+\sqrt{117}$, or about 28.4 units

p. 102 #11a ✓

Chapter 2 Review **Question 11** **Page 102**

a) Find the slopes of the sides of ΔDEF.

$$m_{DE} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-7}{-4-(-2)} = \frac{-5}{-2} = \frac{5}{2}$$

$$m_{EF} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-2}{6-(-4)} = \frac{-4}{10} = -\frac{2}{5}$$

$m_{DF} =$

$\because m_{DE} = \frac{-1}{m_{EF}}$

$\therefore DE \perp EF$

$\therefore \Delta DEF$ is right.

The slopes are negative reciprocals. DE is perpendicular to EF. Therefore, ΔDEF is a right triangle with $\angle DEF = 90°$.

p. 102 #12abc

Chapter 2 Review **Question 12** **Page 102**

a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(65 - 45)^2 + (40 - 60)^2}$

$= \sqrt{20^2 + (-20)^2}$

$= \sqrt{800}$

$\doteq 28.3$

The distance from A to B is about 28.3 km.

b) $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{45 + 65}{2}, \frac{60 + 40}{2}\right)$

$= (55, 50)$

c) Find the equation of line representing the branch pipeline.

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{40 - 60}{65 - 45}$

$= \frac{-20}{20}$

$= -1$

The slope of the line representing the pipeline is -1. The slope of the branch pipeline is 1. Substitute $m = 1$ and the coordinates of the midpoint to find b.

$y = mx + b$

$50 = 1(55) + b$

$-5 = b$

The equation of the line representing the branch line is $y = x - 5$. Check if the given point satisfies the equation.

L.S. $= y$ **R.S.** $= x - 5$

$= 54$ $= 63 - 5$

$= 58$

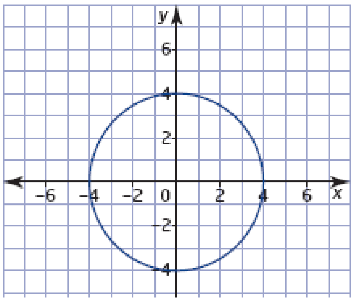
L.S. ≠ R.S.

Point C is not on the branch pipeline.

p. 102 #14ab

Chapter 2 Review **Question 14 Page 102**

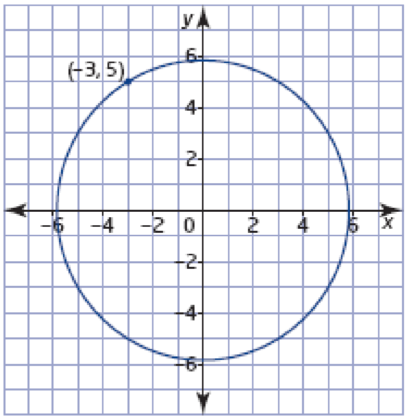
a) $x^2 + y^2 = 16$



b) $x^2 + y^2 = r^2$

$(-3)^2 + 5^2 = r^2$

$34 = r^2$



An equation for the circle is $x^2 + y^2 = 34$.

p. 103 #15

a) $x^2+y^2=r^2$

$x^2+y^2=\left(4\frac{1}{2}\right)^2 \rightarrow \left(\frac{9}{2}\right)^2$

$x^2+y^2=\frac{81}{4}$

b) $x^2+y^2=r^2$

$d=14$

$\therefore r=7$

$x^2+y^2=\left(\cancel{\frac{14}{2}}\right)^2 \; 7^2$

$x^2+y^2=49$

c) $x^2+y^2=r^2$

$x^2+y^2=\left(\sqrt{12}\right)^2$

$x^2+y^2=12$

d) $x^2+y^2=r^2$

$4^2+7^2=r^2$

$16+49=$

$65=r^2$

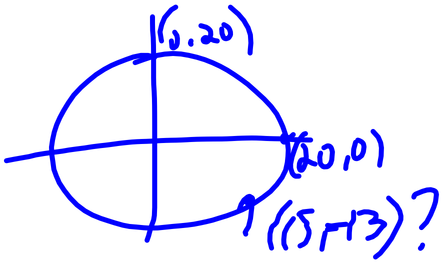
An equation for the circle is $x^2+y^2=65$.

p. 103 #18 ✓

Chapter 2 Review **Question 18** **Page 103**

The radius of the signal area is 20 km. Check that the cell phone user is within the circular area.

$$r_1 = \sqrt{x^2 + y^2}$$
$$= \sqrt{13^2 + 15^2}$$
$$= \sqrt{169 + 225}$$
$$= \sqrt{394}$$
$$\doteq 19.8$$



The user is less than 20 km from the tower. Signals can be received.

$$LS = x^2 + y^2$$
$$= (15)^2 + (-13)^2$$
$$= 225 + 169$$
$$= 394$$

$RS = 400$ $\therefore x^2 + y^2 = 20^2$

$x^2 + y^2 = 400$

$\therefore LS < RS$

$\therefore (15, -13)$ is inside the circle.

Today's solutions from p.104

Chapter 2 Chapter Test Question 5 Page 104

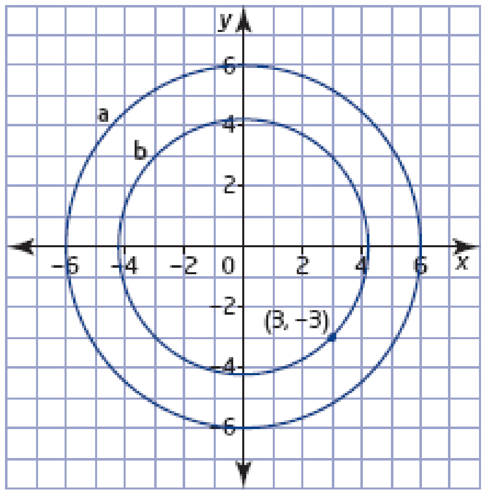
a) $x^2 + y^2 = 36$

b) $x^2 + y^2 = r^2$

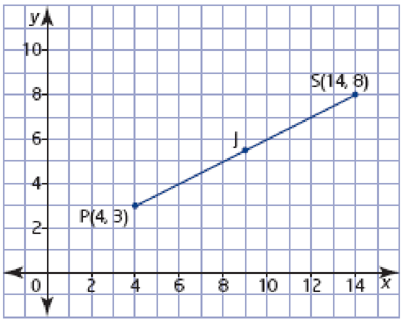
$3^2 + (-3)^2 = r^2$

$18 = r^2$

An equation for the circle is $x^2 + y^2 = 18$.



Chapter 2 Chapter Test **Question 7** **Page 104**



a) $\text{PS} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(14 - 4)^2 + (8 - 3)^2}$

$= \sqrt{10^2 + 5^2}$

$= \sqrt{125}$

$\doteq 11.2$

The schools are about 11.2 km apart.

b) $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{4 + 14}{2}, \frac{3 + 8}{2}\right)$

$= (9, 5.5)$

The coordinates of Jason's home are (9, 5.5).

c) Answers may vary. For example:

Any point on the perpendicular bisector of PS will be equidistant from the two schools.

d) Find the slope of PS.

$m_{\text{PS}} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8 - 3}{14 - 4}$

$= \frac{5}{10}$

$= \frac{1}{2}$

The slope of the right bisector of PS is –2. Substitute $m = -2$ and the coordinates of the midpoint to find b.

$y = mx + b$

$5.5 = -2(9) + b$

$5.5 = -18 + b$

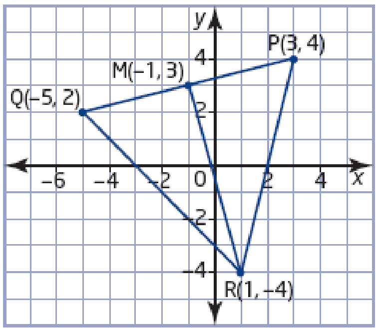
$23.5 = b$

$y = -2x + 23.5$

The equation of the right bisector of PS is $y = -2x + 23.5$.

Chapter 2 Chapter Test **Question 9** **Page 104**

a)



b) Find the coordinates of the midpoint, M, of QP.

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-5 + 3}{2}, \frac{2 + 4}{2}\right)$$
$$= (-1, 3)$$

Find the slope of the median.

$$m_{MR} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-4 - 3}{1 - (-1)}$$
$$= \frac{-7}{2}$$
$$= -\frac{7}{2}$$

Substitute $m = -\frac{7}{2}$ and the coordinates of one endpoint, say $(-1, 3)$, to find b.

$$y = mx + b$$
$$3 = -\frac{7}{2}(-1) + b$$
$$3 = \frac{7}{2} + b$$
$$-\frac{1}{2} = b$$

The equation of the median from vertex R is $y = -\frac{7}{2}x - \frac{1}{2}$.

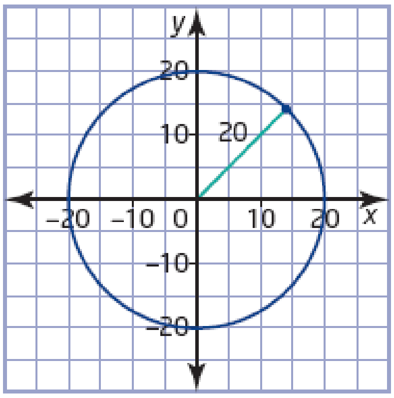
c) Find the slope of side QP.

$$m_{QP} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4 - 2}{3 - (-5)}$$
$$= \frac{2}{8}$$
$$= \frac{1}{4}$$

The slope of QP is not the negative reciprocal of the slope of RM. The median is not an altitude.

Chapter 2 Chapter Test Question 13 Page 105

a)



b) The boundary of the area is described by the equation $x^2 + y^2 = 400$.

c)

$$r_1 = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-8)^2 + 16^2}$$
$$= \sqrt{64 + 256}$$
$$= \sqrt{320}$$
$$\doteq 17.9$$

$$r_1 = \sqrt{x^2 + y^2}$$
$$= \sqrt{4^2 + 20^2}$$
$$= \sqrt{16 + 400}$$
$$= \sqrt{416}$$
$$\doteq 20.4$$

Since $r_1 < 20$, Arif is inside the circle.

Since $r_1 > 20$, Diane is outside the circle.

Arif is in range, but Diane is not.

d) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(-8 - 4)^2 + (16 - 20)^2}$$
$$= \sqrt{(-12)^2 + (-4)^2}$$
$$= \sqrt{160}$$
$$\doteq 12.6$$

Diane and Arif are about 12.6 km apart. They are within range of each other.