

## CHAPTER 2 and 3 EXAM REVIEW- Extended Solutions

(Revised Fall 2016)

1. midpoint JK( $-1, \frac{3}{2}$ ), midpoint KL( $0, -\frac{5}{2}$ ), midpoint JL( $2, 0$ );

$$|KL| = 3\sqrt{5} \text{ units}, |JL| = 2\sqrt{17} \text{ units}, |JK| = \sqrt{41} \text{ units.}$$

It is scalene since  $|KL| \neq |JL| \neq |JK|$

2. a)

$$M_{KL} = \left(0, -\frac{5}{2}\right)$$

find  $b$

$$4 = 6.5(1) + b$$

slope of JM

$$-2.5 = b$$

$$m_{JM} = \frac{4 - (-2.5)}{1 - 0} \\ = 6.5$$

equation

$$y = 6.5x - 2.5$$

- b)

$$M_{JL} = (2, 0)$$

slope of KM<sub>JL</sub>

$$m_{KM} = \frac{-1 - 0}{-3 - 2} \\ = 0.2$$

find  $b$

$$0 = 0.2(2) + b$$

$$-0.4 = b$$

equation

$$y = 0.2x - 0.4$$

- c)

$$M_{JL} = (2, 0)$$

find  $b$  (using M)

slope of JL

$$0 = \frac{1}{4}(2) + b$$

$$m_{JL} = \frac{-4 - 4}{3 - 1} \\ = -4$$

$$-\frac{1}{2} = b$$

slope of right bisector

state the equation

$$m = \frac{1}{4}$$

$$y = \frac{1}{4}x - \frac{1}{2}$$

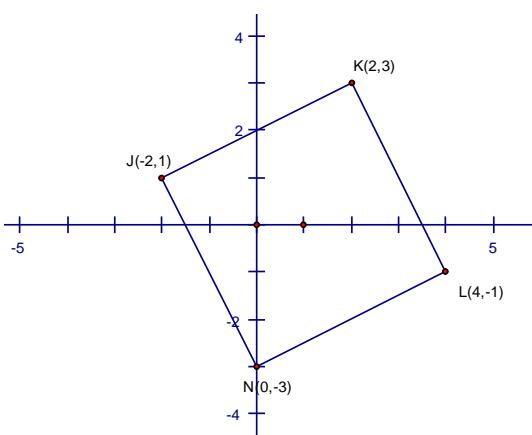
3. Find the length of AC and the length of BC

$$\begin{aligned} |AC| &= \sqrt{(41-10)^2 + (19-63)^2} \\ &= \sqrt{(-31)^2 + (45)^2} \\ &= \sqrt{961 + 2025} \\ &= \sqrt{2986} \\ &\doteq 54.64 \end{aligned}$$

$$\begin{aligned} |BC| &= \sqrt{(87-41)^2 + (30-18)^2} \\ &= \sqrt{46^2 + 12^2} \\ &= \sqrt{2116 + 144} \\ &= \sqrt{2260} \\ &\doteq 47.54 \end{aligned}$$

Fire station B is closer since  $|BC| < |AC|$

- 4.



$$m_{JK} = \frac{1}{2} \quad m_{KL} = -2 \quad m_{LN} = \frac{1}{2} \quad m_{JN} = -2$$

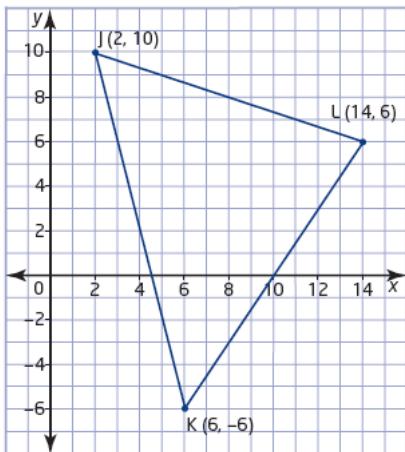
Adjacent sides have negative reciprocal slopes, so adjacent sides are perpendicular. That means this quadrilateral is a rectangle because all angles are right angles.

Check if it is a square.

$$\begin{aligned} |JK| &= \sqrt{4^2 + 2^2} & |JN| &= \sqrt{2^2 + 4^2} \\ &= \sqrt{20} & &= \sqrt{20} \end{aligned}$$

Since two adjacent sides are equal, all four sides must be equal (because of the right angles). So this is a square.

5. a)



b) M(4, 2) and N(8, 8)

c)

$$\begin{aligned}|MN| &= \sqrt{(8-4)^2 + (8-2)^2} & |KL| &= \sqrt{(14-6)^2 + (6-(-6))^2} \\&= \sqrt{4^2 + 6^2} &&= \sqrt{8^2 + 12^2} \\&= \sqrt{52} &&= \sqrt{208} \\&= \sqrt{4}\sqrt{13} &&= \sqrt{16}\sqrt{13} \\&= 2\sqrt{13} &&= 4\sqrt{13}\end{aligned}$$

Since  $|KL| = 2|MN|$ , MN is half of KL.

d)

$$\begin{aligned}m_{MN} &= \frac{8-2}{8-4} \\&= \frac{6}{4} \\&= \frac{3}{2}\end{aligned}$$

$$\begin{aligned}m_{KL} &= \frac{6-(-6)}{14-6} \\&= \frac{12}{8} \\&= \frac{3}{2}\end{aligned}$$

Since  $m_{MN} = m_{KL}$ , MN and KL are parallel.OR  $\because m_{MN} = m_{KL} \therefore MN \parallel KL$ 

6. Find the equation of the right bisector

$$\begin{aligned}M_{QR} &= \left( \frac{-2+4}{2}, \frac{5+1}{2} \right) \\&= (1, 3)\end{aligned}$$

$$\begin{aligned}m_{QR} &= \frac{5-1}{-2-4} \\&= \frac{-2}{3}\end{aligned}$$

$$\begin{aligned}&\text{slope of} \\&\text{right bisector} \\&m = \frac{3}{2}\end{aligned}$$

$$\begin{aligned}y &= mx + b \\3 &= \frac{3}{2}(1) + b \\b &= \frac{3}{2}\end{aligned}$$

The equation of the right bisector is  $y = \frac{3}{2}x + \frac{3}{2}$ 

Check if P(-3, -2) is on the line.

$$\begin{aligned}LS &= y & RS &= \frac{3}{2}x + \frac{3}{2} \\LS &= -2 & RS &= \frac{3}{2}(-3) + \frac{3}{2} \\&&&RS = -3\end{aligned}$$

LS and RS are not equal so the point is not on the right bisector.

7. a)  $x^2 + y^2 = 49$       b)  $x^2 + y^2 = 61$       c)  $x^2 + y^2 = 67$ 

8.

$$r^2 = 16$$

 $r = \sqrt{16}$  The radius is 4 units.

$$r = 4$$

9. The centroid is the point where the three medians of a triangle intersect.  
Determine the equation of two of the medians of the triangle  
and then find the point of intersection of these two lines.

10. Use the slopes to determine if any of the sides are perpendicular.

$$\begin{aligned} m_{DE} &= \frac{4-14}{8-2} & m_{EF} &= \frac{10-4}{18-8} \\ &= \frac{-10}{6} & &= \frac{6}{10} \\ &= \frac{-5}{3} & &= \frac{3}{5} \end{aligned}$$

Since the slopes of  $DE$  and  $EF$  are negative reciprocals,  $DE \perp EF$ . Hence,  $\triangle DEF$  is a right triangle.

11.

$$M = \left( \frac{\frac{-1}{2} + 3}{2}, \frac{2 + \frac{2}{3}}{2} \right)$$

$$M = \left( \frac{\frac{5}{2}}{2}, \frac{\frac{8}{3}}{2} \right)$$

$$M = \left( \frac{5}{2} \times \frac{1}{2}, \frac{8}{3} \times \frac{1}{2} \right)$$

$$M = \left( \frac{5}{4}, \frac{4}{3} \right)$$

12. A parallelogram.

13. The equation of the right bisector of  $JL$

$$M_{JL} = (3, 1)$$

$$m_{JL} = \frac{-1}{2} \quad \text{Slope} = 2$$

$$\begin{aligned} y &= mx + b \\ 1 &= 2(3) + b \quad y = 2x - 5 \\ -5 &= b \end{aligned}$$

The equation of the right bisector of  $JK$

$$M_{JK} = (5, 5)$$

$$m_{JK} = \frac{3}{4} \quad \text{Slope} = -\frac{4}{3}$$

$$\begin{aligned} y &= mx + b \\ 5 &= \frac{-4}{3}(5) + b \quad y = -\frac{4}{3}x + \frac{35}{3} \\ \frac{35}{3} &= b \end{aligned}$$

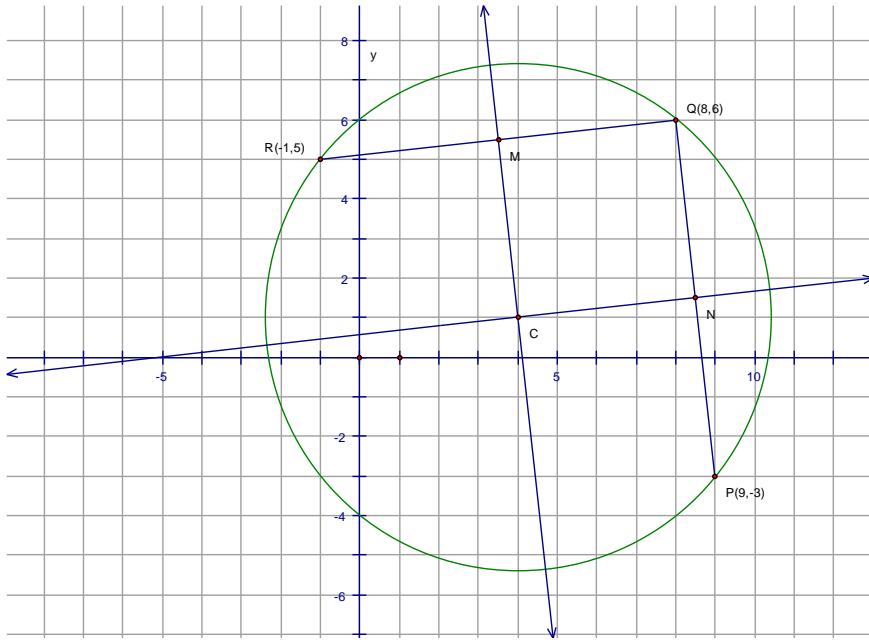
The equation of the right bisector of  $LK$

$$M_{LK} = (7, 4)$$

$$m_{LK} = 2 \quad \text{Slope} = -\frac{1}{2}$$

$$\begin{aligned} y &= mx + b \\ 4 &= \frac{-1}{2}(7) + b \quad y = -\frac{1}{2}x + \frac{15}{2} \\ \frac{15}{2} &= b \end{aligned}$$

14. Find the equation of the right bisector of two of the “chords” of the circle, then find where they cross.



Equation of right bisector of PQ

$$N = \left( \frac{9+8}{2}, \frac{-3+6}{2} \right) \\ = \left( \frac{17}{2}, \frac{3}{2} \right)$$

$$m_{PQ} = \frac{-3-6}{9-8} = -9 \\ slope = \frac{1}{9}$$

$$y = mx + b \\ \frac{3}{2} = \frac{1}{9} \left( \frac{17}{2} \right) + b \\ \frac{3}{2} = \frac{17}{18} + b \\ \frac{27}{18} - \frac{17}{18} = b \\ \frac{10}{18} = b \\ \frac{5}{9} = b$$

$$y = \frac{1}{9}x + \frac{5}{9}$$

Equation of right bisector of RQ

$$M = \left( \frac{8-1}{2}, \frac{6+5}{2} \right) \\ = \left( \frac{7}{2}, \frac{11}{2} \right)$$

$$m_{RQ} = \frac{6-5}{8-(-1)} = \frac{1}{9} \\ slope = -9$$

$$y = mx + b \\ \frac{11}{2} = -9 \left( \frac{7}{2} \right) + b \\ \frac{11}{2} = \frac{-63}{2} + b \\ \frac{74}{2} = b \\ 37 = b$$

$$y = -9x + 37$$

Find the point of intersection of the two right bisectors.

$$\frac{1}{9}x + \frac{2}{9} = -9x + 37$$

$$\frac{1}{9}x + 9x = 37 - \frac{2}{9}$$

$$\frac{1}{9}x + \frac{81}{9}x = \frac{333}{9} - \frac{5}{9}$$

$$\frac{82}{9}x = \frac{328}{9}$$

$$82x = 328$$

$$x = 4$$

$$sub \quad x = 4 \quad int o \quad y = -9x + 37$$

$$y = -9(4) + 37$$

$$y = -36 + 37$$

$$y = 1$$

The centre of the circle is at (4, 1)

Another solution for #14.

### #14 Units 2/3 HPM 2D1



chords QR and PQ are made

right bisector of QR:

$$M_{QR} \left( \frac{-1+8}{2}, \frac{5+6}{2} \right)$$

$$= M(3.5, 5.5)$$

$$m_{QR} = \frac{6-5}{8-(-1)} = \frac{1}{9}$$

$$\therefore m_{\perp} = -9$$

$$y = m_{\perp}x + b$$

$$5.5 = -9(3.5) + b$$

$$\therefore b = 37$$

$$\text{Hence, it is } y = -9x + 37$$

right bisector of PQ :

$$M_{PQ} \left( \frac{8+9}{2}, \frac{6+(-3)}{2} \right)$$

$$= M(8.5, 1.5)$$

$$m_{PQ} = \frac{6-(-3)}{8-9} = \frac{9}{-1} = -9$$

$$\therefore m_{\perp} = \frac{1}{9}$$

$$y = m_{\perp}x + b$$

$$y = \frac{1}{9}x + b$$

$$1.5 = \frac{1}{9}(8.5) + b$$

$$\therefore b = 0.5 \leftarrow \text{NOTICE}$$

$$\text{Hence, it is } y = \frac{1}{9}x + 0.5$$

Now find solution of system

$$y = -9x + 37 \quad ①$$

$$y = \frac{1}{9}x + 0.5 \quad ②$$

Sub ② into ① :

$$\frac{1}{9}x + 0.5 = -9x + 37$$

mult by 9

$$x + 5 = -81x + 333$$

$$82x = 328$$

$$\therefore x = 4$$

sub into ① :

$$y = -9(4) + 37$$

$$y = -36 + 37$$

$$y = 1$$

HENCE, the centre  
is (4, 1)