Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 198-199 #1c, 2ac, 3, 4ab, 5–8 [11]

6, 7

Day 1 Review Homework includes:

pp. 202-203 #1 – 12, 13 – 17, 19 – 23

5, 6, 11, 16, 15, 14, 17, 20, 23

HIGHLY RECOMMENDED

Worksheet on Class Website:

"Word Problems Involving Quadratics" #1 – 10

Day 2 Review Homework includes:

p. 204 #1 – 9

pp. 202 #5

5. The height, $h(t)$, in metres, of the trajectory of a football is given by $h(t) = 2 + 28t - 4.9t^2$, where t is the time in flight, in seconds. Determine the maximum height of the football and the time when that height is reached. $\rightarrow$ ∴ vertex

$$h(t) = -4.9t^2 + 28t + 2$$

$$t = \frac{-b}{2a}$$

$$= \frac{-(28)}{2(-4.9)}$$

$$\approx 2.857$$

$$t \doteq 2.9$$

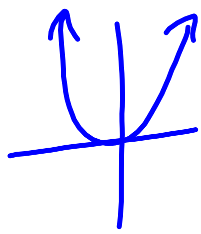
$$\therefore h(2.9) \doteq -4.9(2.9)^2 + 28(2.9) + 2$$

$$\doteq 41.99$$

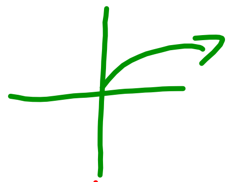
$$\doteq 42\text{ m}$$

6. Describe the relationship between $f(x) = x^2$, $g(x) = \sqrt{x}$, and $h(x) = -\sqrt{x}$.

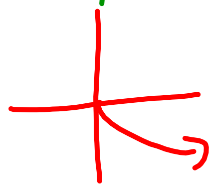
$f(x)$



$g(x)$:



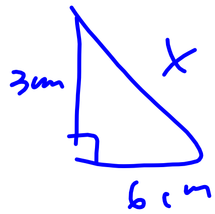
$h(x)$:



pp. 202 #11

11. What is the perimeter of a right triangle with legs 6 cm and 3 cm? Leave your answer in simplest radical form.

(P7)



$x^2 = 3^2 + 6^2$

$= 9 + 36$

$= 45$

$x = \sqrt{45}$

$= \sqrt{9}\sqrt{5}$

$= 3\sqrt{5}$

$P = 6 + 3 + x$

$= 9 + 3\sqrt{5}$ cm

pp. 203 #13

13. The population of a Canadian city is modelled by $P(t) = 12t^2 + 800t + 40\ 000$, where t is the time in years. When $t = 0$, the year is 2007.

a) According to the model, what will the population be in 2020?

b) In what year is the population predicted to be 300 000?

a) in 2020 $t = 2020 - 2007$

$= 13$

$P(13) = 12(13)^2 + 800(13) + 40\ 000$

$= 52\ 428$

b) Let $P(t) = 300\ 000$

$300000 = 12t^2 + 800t + 40\ 000$

$0 = 12t^2 + 800t + 40\ 000 - 300\ 000$

$0 = 12t^2 + 800t - 260\ 000$

$a = 12 \quad b = 800 \quad c = -260\ 000$

$$t = \frac{-800 \pm \sqrt{800^2 - 4(12)(-260\ 000)}}{2(12)}$$

$$= \frac{-800 \pm \sqrt{13\ 120\ 000}}{24}$$

$t \doteq -184.2$ or $t \doteq 117.5$

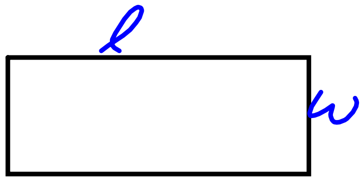
inadmissible? $\therefore$ year $2007 + 117.5$

$= 2124.5$

$\therefore$ during the year 2124 the population will reach 300 000.

pp. 203 #14

14. A rectangular field with an area of 8000 m^2 is enclosed by 400 m of fencing. Determine the dimensions of the field to the nearest tenth of a metre.



Let l and w represent the length and width of the field respectively, in m.

$400 = 2l + 2w$

$200 = l + w$

$200 - w = l$

$A = lw$

$8000 = lw$

$8000 = (200-w)(w)$

$8000 = 200w - w^2$

* Solve. $w^2 - 200w + 8000 = 0$

$a = 1 \quad b = -200 \quad c = 8000$

$w^2 - 200w + 8000 = 0$

$w^2 - 200w + 100^2 - 100^2 + 8000 = 0$

$(w-100)^2 - 10000 + 8000 = 0$

$(w-100)^2 - 2000 = 0$

$(w-100)^2 = 2000$

$w - 100 = \pm\sqrt{2000}$

$w = 100 \pm \sqrt{2000}$

$w \doteq 100 + 44.72$ or $w \doteq 100 - 44.72$

$\doteq 144.72$ $\qquad \doteq 55.28$

if $w \doteq 144.72$

$l = 200 - w$

$\doteq 200 - 144.72$

$\doteq 55.28$ m

if $w = 55.28$

$\therefore l = 144.72$ m

the dimensions of the field are 55.28 m by 144.72 m.

pp. 203 #15

15. The height, $h(t)$, of a projectile, in metres, can be modelled by the equation $h(t) = 14t - 5t^2$, where t is the time in seconds after the projectile is released. Can the projectile ever reach a height of 9 m? Explain.

Can $h(t) = 9$

$9 = 14t - 5t^2$

$0 = -5t^2 + 14t - 9$

$b^2 - 4ac$

$= (14)^2 - 4(-5)(-9)$

$= 196 - 180$

$= 16$

$\therefore b^2 - 4ac > 0$

$\therefore$ the projectile can reach 9m.

pp. 203 #16

16. Determine the values of k for which the function $f(x) = 4x^2 - 3x + 2kx + 1$ has two zeros. Check these values in the original equation.

$$= 4x^2 - 3x + 2kx + 1$$

$$f(x) = 4x^2 + x(-3+2k) + 1$$

$$\therefore a = 4 \quad b = -3+2k \quad c = 1$$

$$b^2 - 4ac > 0$$

$$(-3+2k)^2 - 4(4)(1) > 0$$

$$9 - 12k + 4k^2 - 16 > 0$$

$$4k^2 - 12k - 7 > 0$$

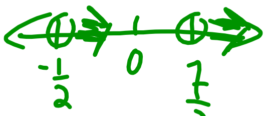
$$(2k+1)(2k-7) > 0$$

Option 1

$2k+1 > 0$ AND $2k-7>0$

$2k > -1$ $2k > 7$

$k > -\frac{1}{2}$ $k > \frac{7}{2}$



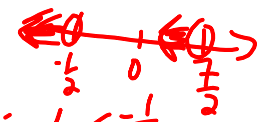
$\therefore k > \frac{7}{2}$

Option 1

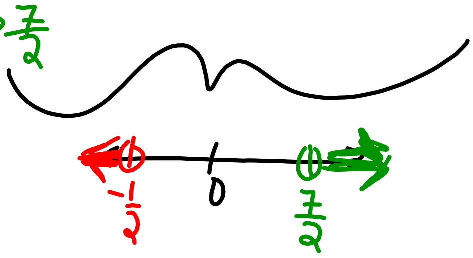
$2k+1<0$ AND $2k-7<0$

$2k<-1$ $2k<7$

$k<-\frac{1}{2}$ $k<\frac{7}{2}$



$\therefore k < -\frac{1}{2}$



$\therefore$ if $k < -\frac{1}{2}$ or $k > \frac{7}{2}$

$f(x) = 4k^2 - 3x + 2kx + 1$ has 2 zeros.

pp. 203 #17

only 1 point.

17. Determine the break-even points of the profit function $P(x) = -2x^2 + 7x + 8$, where x is the number of dirt bikes produced, in thousands.

Let $P(x)=0$

$0=-2x^2+7x+8$

$x=\frac{-7\pm\sqrt{(7)^2-4(-2)(8)}}{2(-2)}$

$=\frac{-7\pm\sqrt{49+64}}{-4}$

$=\frac{-7\pm\sqrt{113}}{-4}$

$x=\frac{-7+\sqrt{113}}{-4}$ or $x=\frac{-7-\sqrt{113}}{-4}$

$\doteq -0.90$ $\qquad\qquad \doteq 4.4075$

inadmissible $\qquad \therefore 4407.5$ bikes

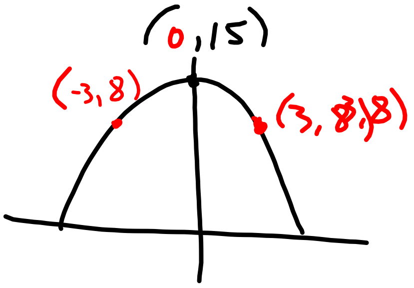
$\therefore 4408$ bikes must be produced to break even.

pp. 203 #20

20. An engineer is designing a parabolic arch. The arch must be 15 m high, and 6 m wide at a height of 8 m.

a) Determine a quadratic function that satisfies these conditions.

b) What is the width of the arch at its base?



$y = a(x-0)^2 + 15$

$y = ax^2 + 15$

$8 = a(3)^2 + 15$

$8 = 9a + 15$

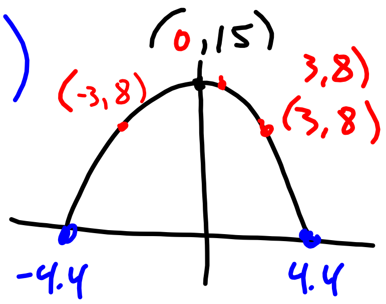
$8 - 15 = 9a$

$\frac{-7}{9} = a$

$\therefore y = \frac{-7}{9}x^2 + 15$

is an equation.

b)



let $y = 0$

$0 = -\frac{7}{9}x^2 + 15$

$\frac{7}{9}x^2 = 15$

$x^2 = \frac{9}{7}(15)$

$= \frac{135}{7}$

$x = \pm\sqrt{\frac{135}{7}}$

$\doteq \pm 4.39$

$\doteq \pm 4.4$

$\therefore$ width $\doteq 2(4.4)$

$\doteq 8.8$

$\therefore$ the width of the arch is about 8.8 m.

pp. 203 #23

23. a) Will the parabola defined by $f(x) = x^2 - 6x + 9$ intersect the line $g(x) = -3x - 5$? Justify your answer.

b) Change the slope of the line so that it will intersect the parabola in two locations.

a) let $f(x) = g(x)$

$x^2 - 6x + 9 = -3x - 5$

$x^2 - 6x + 9 + 3x + 5 = 0$

$x^2 - 3x + 14 = 0$

check discriminant

$b^2 - 4ac$

$= (-3)^2 - 4(1)(14)$

$= 9 - 56$

$= -47$

$\therefore b^2 - 4ac < 0$

$\therefore f(x)$ does not intersect $g(x)$

b) let $g(x) = mx - 5$

$x^2 - 6x + 9 - mx + 5 = 0$

$x^2 + x(-6 - m) + 14 = 0$

Now $b^2 - 4ac > 0$ for 2 intersections

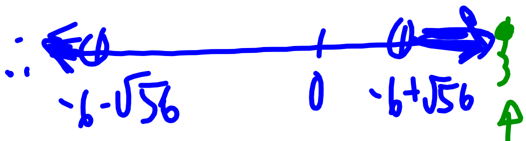
$(-6 - m)^2 - 4(1)(14) > 0$

$36 + 12m + m^2 - 56 > 0$

$m^2 + 12m - 20 > 0$

don't worry from here, but I got

$m = -6 \pm \sqrt{56}$

$\therefore$ 

Solution ex let $m = 3$

$\therefore$ ask me in class.

p. 198 #4a

4. Determine the point(s) of intersection of each pair of functions.

K a) $f(x) = -2x^2 - 5x + 20$, $g(x) = 6x - 1$

Let $6x - 1 = -2x^2 - 5x + 20$

$2x^2 + 5x + 6x - 1 - 20 = 0$

$2x^2 + 11x - 21 = 0$

$(2x - 3)(x + 7) = 0$

$\therefore x = \frac{3}{2}$ or $x = -7$

$g\left(\frac{3}{2}\right) = \cancel{6}^{3}\left(\frac{3}{\cancel{2}}\right) - 1$

$= 8$

$\therefore \left(\frac{3}{2}, 8\right)$

$g(-7) = 6(-7) - 1$

$= -43$

$\therefore (-7, -43)$

p. 198 #5

5. An integer is two more than another integer. Twice the larger integer is one more than the square of the smaller integer. Find the two integers.

Let x represent the larger integer.
Let y represent the smaller integer.

$2x = y^2 + 1$

$x = y + 2$

$2(y + 2) = y^2 + 1$

$2y + 4 = y^2 + 1$

$0 = y^2 - 2y - 4 + 1$

$0 = y^2 - 2y - 3$

$0 = (y - 3)(y + 1)$

$\therefore y = 3$ or $y = -1$

if $y = 3$

$x = 3 + 2$

$= 5$

if $y = -1$

$x = -1 + 2$

$= 1$

$\therefore$ the numbers are 5 and 3

or 1 and -1.

p. 199 #6

6. The revenue function for a production by a theatre group is $R(t) = -50t^2 + 300t$, where t is the ticket price in dollars. The cost function for the production is $C(t) = 600 - 50t$. Determine the ticket price that will allow the production to break even.

$P(t) = \text{Revenue} - \text{Cost}$

(Profit) $= -50t^2 + 300t - (600 - 50t)$

(break even) $= -50t^2 + 300t - 600 + 50t$

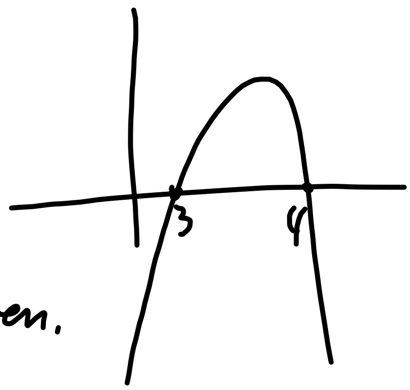
$\hookrightarrow 0 = -50t^2 + 350t - 600$

$= -50(t^2 - 7t + 12)$

$= -50(t-4)(t-3)$

$\therefore t = 4$ or $t = 3$

$\therefore$ if tickets cost \$3 each the production will break even.

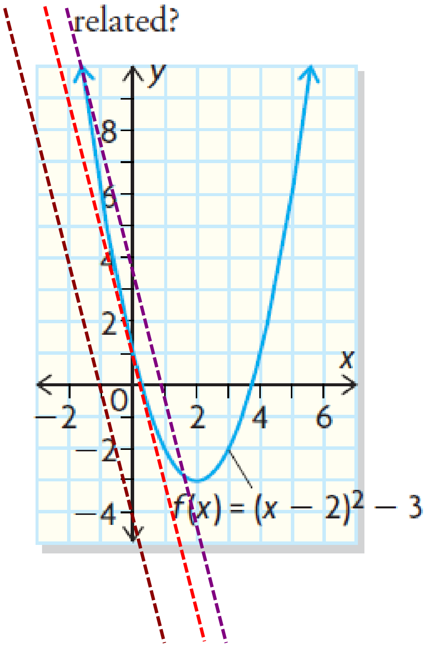


p. 199 #7

7. a) Copy the graph of $f(x) = (x - 2)^2 - 3$. Then draw lines with slope -4 that intersect the parabola at (i) one point, (ii) two points, and (iii) no points.

b) Write the equations of the lines from part (a).

c) How are all of the lines with slope -4 that do not intersect the parabola related?



No P.O.I $y=-4x-6$

1 P.O.I $y=-4x+1$

2 P.O.I. $y=-4x+4$

$y=(x-2)^2-3$ $\quad$ $y=-4x+b$

$(x-2)^2-3=-4x+b$

$x^2-4x+4-3+4x-b=0$

$1x^2 + 0x + 1-b=0$

$ax^2+bx+c=0$

$a=1 \quad b=0 \quad c=1-b$

tangent: $b^2-4ac=0$

$(0)^2-4(1)(1-b)=0$

$-4+4b=0$

$4b=4$

$b=1$

p. 199 #8

8. Determine the value of k such that $g(x) = 3x + k$ intersects the quadratic function $f(x) = 2x^2 - 5x + 3$ at exactly one point.

$$2x^2 - 5x + 3 = 3x + k$$
$$2x^2 - 5x - 3x + 3 - k = 0$$
$$2x^2 - 8x + 3 - k = 0$$
$$\therefore a = 2 \quad b = -8 \quad c = 3 - k$$

$$b^2 - 4ac = 0$$
$$(-8)^2 - 4(2)(3-k) = 0$$
$$64 - 24 + 8k = 0$$
$$40 = -8k$$
$$\frac{40}{-8} = k$$
$$k = -5$$

$\therefore$ if $k = -5$ (or $y = 3x - 5$)

then $g(x)$ intersects $f(x)$ at exactly one point.

Quadratics Review

Date: Mar. 29/17

1. For each function below state the direction of the opening, the vertex, axis of symmetry, max or min value, and the domain and range. Finally, sketch the function.

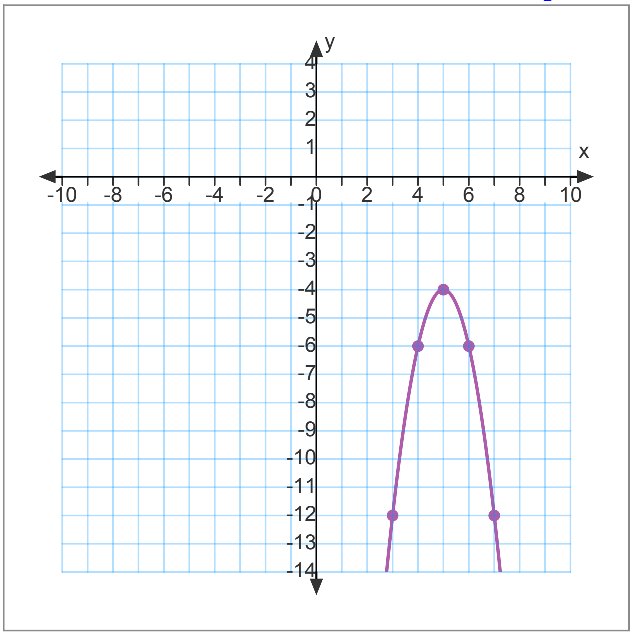
a) $f(x) = -2(x-5)^2 - 4$

down

$v(5,-4)$ A of S: $x=5$

max of -4

D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid y \leq -4\}$



$y = -2(x-5)^2 - 4$

b) $f(x) = 2(x-3)(x+5)$

up if $f(x)=0$

$\hookrightarrow x=3$ or $x=-5$

A of S: $x = \frac{-5+3}{2}$

$x=-1$

vertex: $f(-1) = 2(-1-3)(-1+5)$

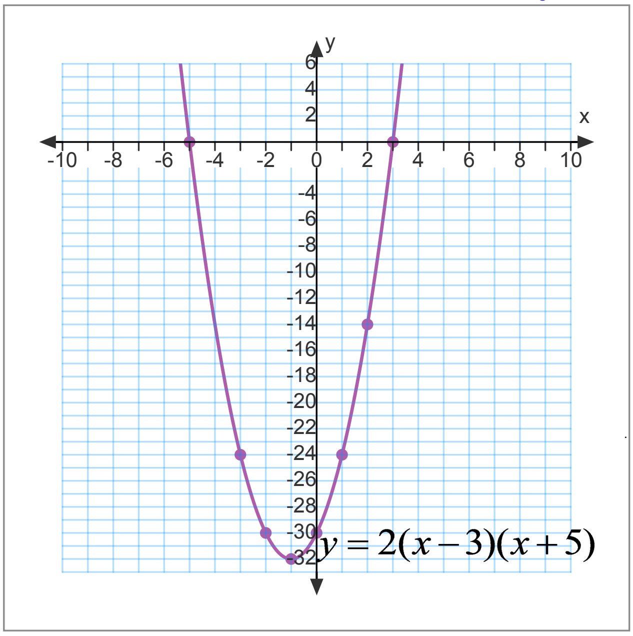
$= 2(-4)(4)$

$= -32$

$\therefore v(-1,-32)$

min value is -32

D: $\{x \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid y \geq -32\}$

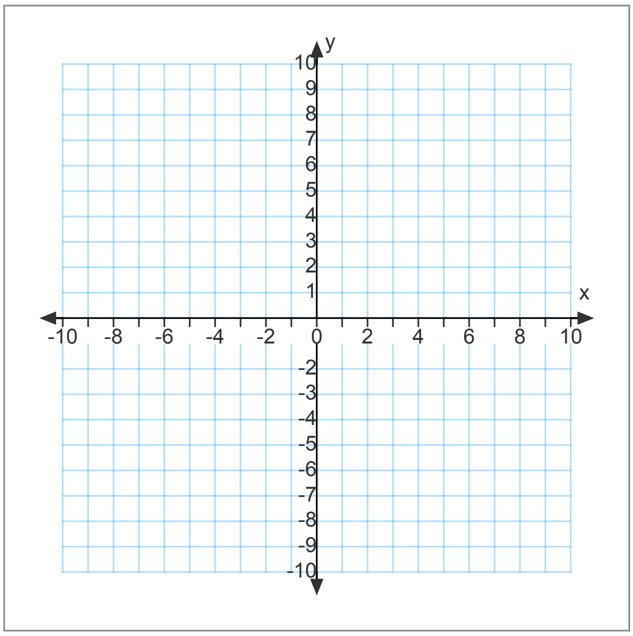


2. a) The height, $h(t)$, in metres, of the tragectory of a football is given by $h(t) = 2 + 28t - 4.9t^2$, where t is the time in flight, in seconds. Determine the maximum height of the football and the time when that height is reached.

b) How long will it take for the ball to hit the ground?

3. a) Determine the inverse of $f(x) = -3(x-4)^2 + 2$

b) Graph $f(x)$ and $f^{-1}(x)$



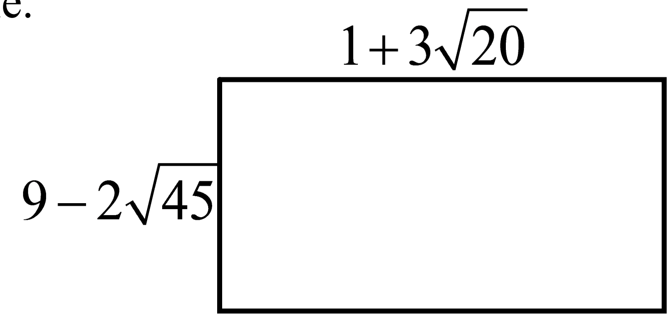
c) Is the inverse a function? Explain using words.

d) State the domain and range of $f(x)$ and $f^{-1}(x)$

4. Express each radical in simplest radical form.

a) $\sqrt{98}$ b) $-5\sqrt{50}$ c) $-2\sqrt{12}+4\sqrt{48}$

5. Determine an expression in lowest terms for the perimeter AND area of the rectangle.



6. a) The height, $h(t)$, of a projectile, in metres, can be modelled by the equation $h(t) = 14t - 5t^2$, where t is the time in seconds after the projectile is released. Can the projectile ever reach a height of 9 m?

b) How long will it take for it to hit the ground?

7. Determine the value(s) for k for which the function has no roots.

$$f(x) = 3x^2 - 4x + k$$

8. Determine the equation of parabola that has roots $\sqrt{5}$ and $-\sqrt{5}$ and goes through point (-1, 6).

9. Solve $f(x) = 3x^2 - 4x + 2$