Today's Learning Goal(s):

By the end of the class, I will be able to:

a) use the Sine Law to solve a triangle that is the **ambiguous** case.

Last day's work: pp. 300-301 #6 – 9ace, 10, 12 [15]
Review p. 304 #1 – 13

p300
6c

p.304
12
13

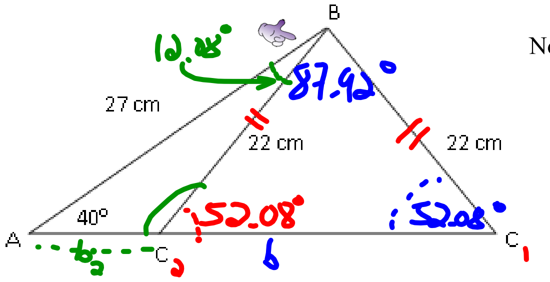
5.6 The Sine Law

Date: May 1/17

Recall: We use the Sine Law when we are have an "opposite pair".
The formula:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex. 1 Consider ΔABC, $\angle A = 40°$, AB=27 cm ($c=27$), and BC = 22 cm ($a=22$). ***Make a sketch.***



Note: There are 2 different ways to sketch ΔABC using this information.
This means there are two possible ways to solve this triangle.
This is the ambiguous case of the Sine Law.

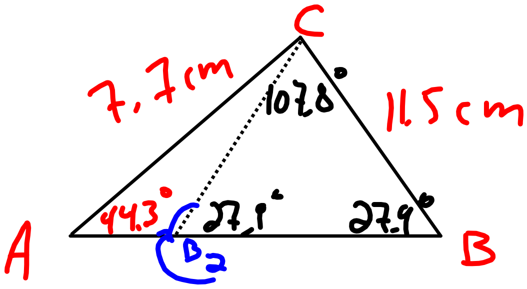
$\angle C = 52.1°, \angle B = 87.9°, b = 34.2$ cm

$\angle C = 127.9°, \angle B = 12.1°, b = 7.2$ cm

$\angle C_1$	$\angle B$	b (AC)
$\frac{\sin C}{27} = \frac{\sin 40°}{22}$ $\sin C = 27 \times \frac{\sin 40°}{22}$ $C = \sin^{-1}\left(27 \times \frac{\sin 40°}{22}\right)$ $\doteq 52.080$ $\doteq 52.08°$	$B \doteq 180° - 40° - 52.08°$ $\doteq 87.92$	$\frac{b}{\sin 87.92°} \doteq \frac{22}{\sin 40°}$ $b \doteq \sin 87.92° \times \frac{22}{\sin 40°}$ $\doteq 34.2033$ $\doteq 34.203$ cm

$\angle C_2$	$\angle B$	b_2
$C_2 \doteq 180° - 52.08°$ $\doteq 127.92°$	$B \doteq 180° - 40° - 127.92°$ $\doteq 12.08°$	$\frac{b_2}{\sin 12.08°} \doteq \frac{22}{\sin 40°}$ $b_2 \doteq \sin 12.08° \times \frac{22}{\sin 40°}$ $\doteq 7.1627$ $\doteq 7.163$ cm

Ex. 2 Solve ΔABC, $\angle A = 44.3°$, a = 11.5 cm, and b = 7.7 cm.

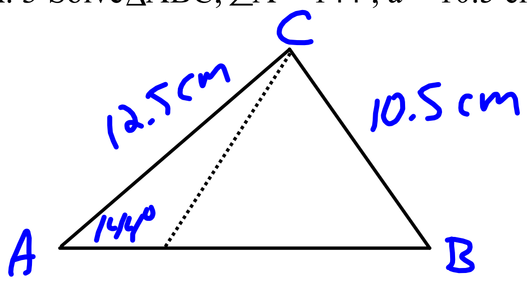


$\angle B= 27.9°, \angle C= 107.8°$, c=15.7 cm

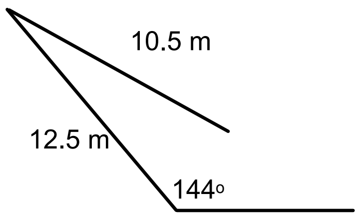
$\angle B= 152.1°, \angle C= -16.4°$

$\angle B$	$\angle C$	c
$\frac{\sin B}{7.7} = \frac{\sin 44.3°}{11.5}$ $B = \sin^{-1}\left(7.7 \times \frac{\sin 44.3°}{11.5}\right)$ $\doteq 27.88$ $\doteq 27.9°$	$C \doteq 180° - 44.3° - 27.9°$ $\doteq 107.8°$	$\frac{c}{\sin 107.8°} \doteq \frac{11.5}{\sin 44.3°}$ $c \doteq \sin 107.8° \times \frac{11.5}{\sin 44.3°}$ $\doteq 15.67$ $\doteq 15.7$ cm
$\angle B_2$	**$\angle C_2$**	
$B_2 \doteq 180° - 27.9°$ $\doteq 152.1°$	$C_2 \doteq 180° - 44.3° - 152.1°$ $\doteq -16.4°$ $\therefore$ only 1 triangle is possible.	

Ex. 3 Solve ΔABC, $\angle A = 144^\circ$, a = 10.5 cm, and b = 12.5 cm.



$\angle B = 44.4^\circ, \angle C = -8.4^\circ$



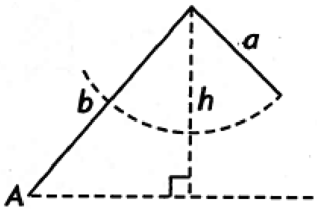
$\angle B$	$\angle C$	
$\frac{\sin B}{12.5} = \frac{\sin 144^\circ}{10.5}$ $\sin B = 12.5 \times \frac{\sin 144^\circ}{10.5}$ $B = \sin^{-1}\left(12.5 \times \frac{\sin 144^\circ}{10.5}\right)$ $\doteq 44.40$ $\doteq 44.4^\circ$	$C \doteq 180^\circ - 144^\circ - 44.4^\circ$ $\doteq -8.4^\circ$	$\therefore$ NO triangles are possible. **See Next Slide for New Summaries**

The ambiguous case arises in a SSA (side, side, angle) triangle. In this situation, depending on the size of the given angle and the lengths of the given sides, the sine law calculation may lead to 0, 1, or 2 solutions.

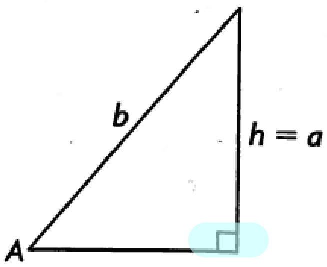
Need to Know

- In the ambiguous case, if $\angle A$, a, and b are given and $\angle A$ is acute, there are four cases to consider. In each case, the height of the triangle is $h = b \sin A$.

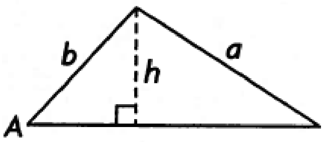
If $\angle A$ is acute and $a < h$, no triangle exists.



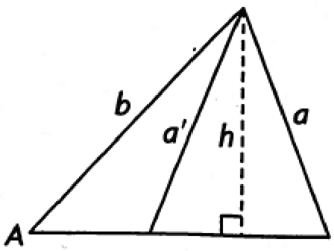
If $\angle A$ is acute and $a = h$, one right triangle exists.



If $\angle A$ is acute and $a > b$, one triangle exists.

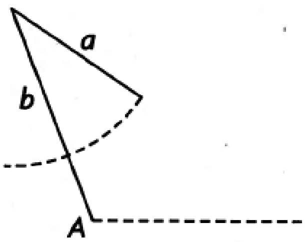


If $\angle A$ is acute and $h < a < b$, two triangles exist.

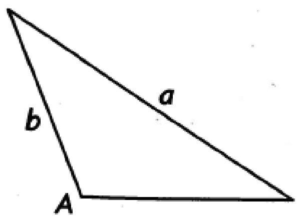


If $\angle A$, a, and b are given and $\angle A$ is obtuse, there are two cases to consider.

If $\angle A$ is obtuse and $a < b$ or $a = b$, no triangle exists.



If $\angle A$ is obtuse and $a > b$, one triangle exists.



Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 300-301 #6 – 9ace, 10, 12 [15]
Review p. 304 #1 – 13

Today's Homework Practice includes:

pp. 318-319 #1, 2, 3a, 4, 5ac, 7 [15,17]