Before we begin, are there any questions from last day's work? 5.3.1

pp.36-40 (1,2)ac,4,5,10,12,13 p.71 14,17

(Tuesday's quiz will be based on the first three lessons...not today's)

Today's Learning Goal(s):

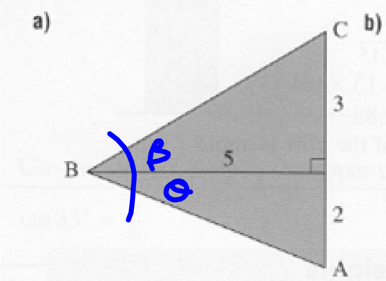
By the end of the class, I will:

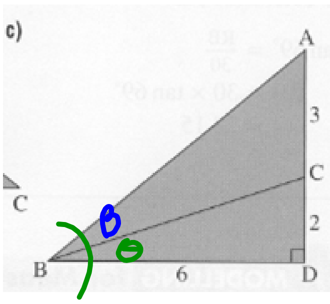
a) understand, that when the diagram is not included, the information may lead to *two different drawings* of the triangle, and is therefore ambiguous.

b) be able to create the diagram and solve for both possible triangles.

p.36 #2

2. Calculate the measure of ∠ABC.





$\tan\theta = \frac{2}{5}$

$\tan\beta = \frac{3}{5}$

$\tan\theta = \frac{2}{6}$

$\theta = \tan^{-1}\left(\frac{2}{6}\right)$

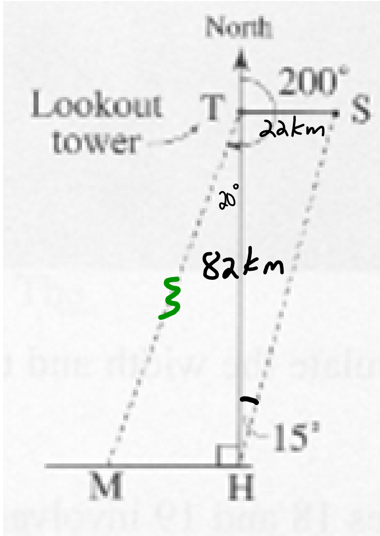
$\doteq$

∠ABD

$\tan B = \frac{5}{6}$

$B =$

13. A person in a lookout tower T reports smoke at S, due east of the tower. She estimates the distance to the smoke is 22 km. A helicopter carrying firefighters lifts off from its base H, 82 km due south of the tower. From the helicopter base, the smoke is on a bearing of 015°. The person in the tower reports a second smoke sighting at M, on a bearing of 200°. From the helicopter base, the second smoke sighting is due west. The helicopter drops the firefighters at S, flies to T to pick up more firefighters, then flies to M. Calculate the total distance the helicopter travels.



$$\text{Distance}_{total} = HS + ST + TM$$

$$\doteq 84.900 + 22 + 87.263$$

$$\doteq 194.163 \text{ km}$$

$$\cos 20° = \frac{82}{TM}$$

$$TM = \frac{82}{\cos 20°}$$

$$\doteq 87.2625$$

$$\doteq 87.263$$

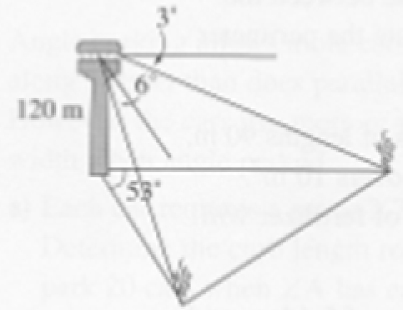
$$HS^2 = 22^2 + 82^2$$

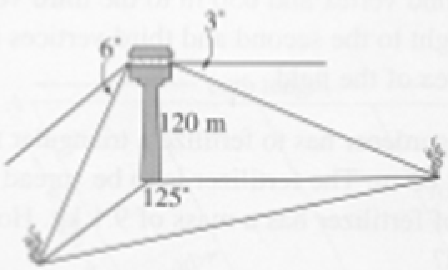
$$= 7208$$

$$\therefore HS \doteq 84.8999$$

$$\doteq 84.900 \text{ km}$$

14. Turn to exercise 12c, page 39. Suppose the smoke sightings are observed in directions that are 53° apart (below left). Calculate the distance between the smoke sightings. The diagram is not to scale.





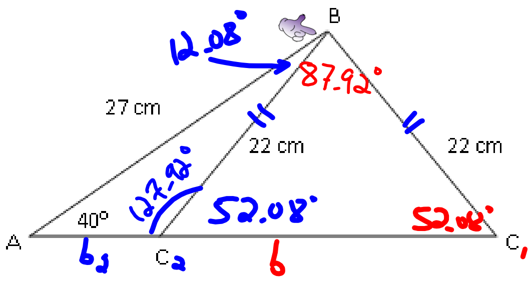
17. Turn to exercise 13 on page 39. Suppose the helicopter flew from the first smoke sighting to the second smoke sighting, not travelling to the lookout tower. Calculate the distance between the two smoke sightings.

5.4.1: The **Ambiguous** Case of the Sine Law

Date: May 1/17

$c = 27\text{ cm}$ $a = 22\text{ cm}$

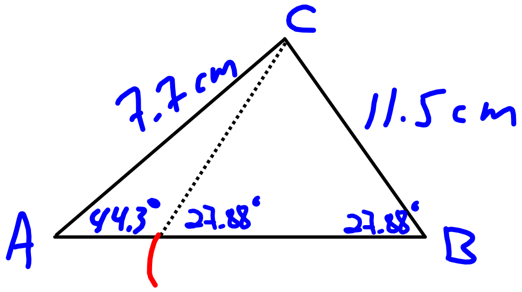
Ex. 1 Consider ΔABC, $\angle A = 40°$, AB=27 cm, and BC = 22 cm.

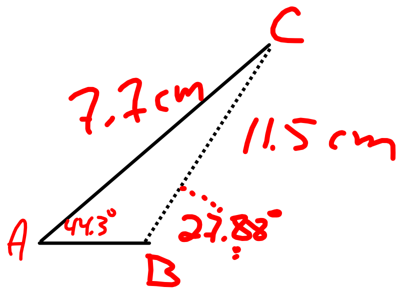


Note: There are 2 different ways to sketch ΔABC using this information. This means there are two possible ways to solve this triangle. This is the ambiguous case of the Sine Law.

$\angle C_1$	$\angle B$	b
$\frac{\sin C_1}{27} = \frac{\sin 40°}{22}$ $\sin C = \frac{27 \times \sin 40°}{22}$ $C = \sin^{-1}\left(\frac{27 \times \sin 40°}{22}\right)$ $\doteq 52.080$ $\doteq 52.08$	$B \doteq 180° - 40° - 52.08$ $\doteq 87.92$	$\frac{b}{\sin 87.92°} \doteq \frac{22}{\sin 40°}$ $b \doteq \sin 87.92° \times \frac{22}{\sin 40°}$ $\doteq 34.2033$ $\doteq 34.203\text{ cm}$
$\angle C_2$	$\angle B$	b_2
$C \doteq 180° - 52.08°$ $\doteq 127.92°$	$B \doteq 180° - 40° - 127.92°$ $\doteq 12.08°$	$\frac{b_2}{\sin 12.08°} \doteq \frac{22}{\sin 40°}$ $b_2 \doteq \sin 12.08° \times \frac{22}{\sin 40°}$ $\doteq 7.1627$ $\doteq 7.163\text{ cm}$

Ex. 2 Solve $\triangle ABC$, $\angle A = 44.3°$, a = 11.5 cm, and b = 7.7 cm.

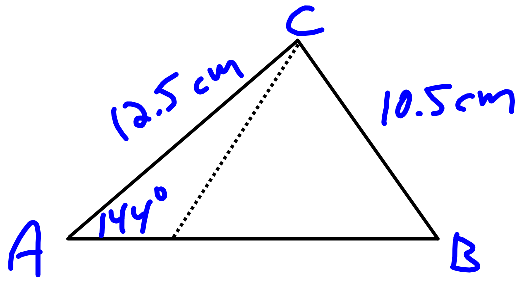


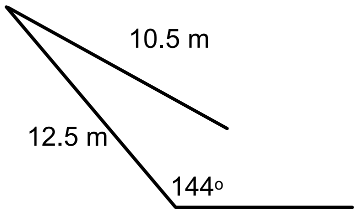


$\angle B$	$\angle C$	c
$\frac{\sin B}{7.7} = \frac{\sin 44.3°}{11.5}$ $B = \sin^{-1}\left(7.7 \times \frac{\sin 44.3°}{11.5}\right)$ $\doteq 27.880$ $\doteq 27.88°$	$C \doteq 180° - 44.3° - 27.88°$ $\doteq 107.82°$	$\frac{c}{\sin 107.82°} \doteq \frac{11.5}{\sin 44.3°}$ $c \doteq \sin 107.82° \times \frac{11.5}{\sin 44.3°}$ $\doteq 15.6758$ $\doteq 15.676$ cm

$\angle B$	$\angle C$	
$\angle B_2 \doteq 180° - 27.88°$ $\doteq 152.12°$	$C \doteq 180° - 44.3° - 152.12$ $\doteq -16.42°$ $\therefore$ only 1 triangle possible	

Ex. 3 Solve $\triangle ABC$, $\angle A = 144°$, a = 10.5 cm, and b = 12.5 cm.





$\angle B$	$\angle C$
$\frac{\sin B}{12.5} = \frac{\sin 144°}{10.5}$ $B = \sin^{-1}\left(12.5 \times \frac{\sin 144°}{10.5}\right)$ $\doteq 44.406$ $\doteq 44.41$	$C \doteq 180° - 144° - 44.41$ $\doteq -8.41°$ $\therefore$ No triangles possible

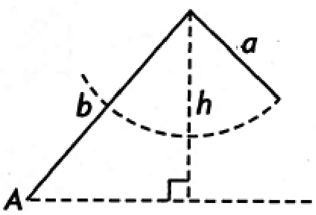
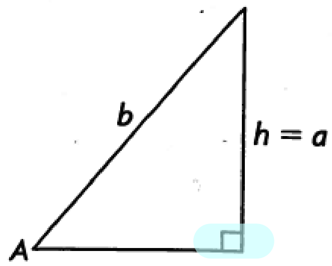
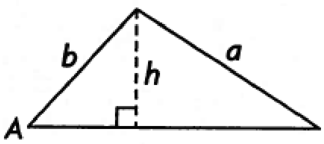
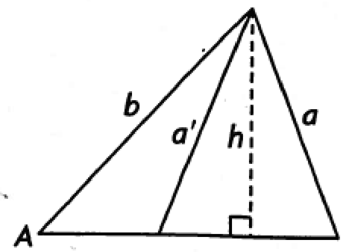
Today's Assigned Work:
5.4.2 2,6,12a
(SWYK 5.1 next class)

See Next Slide for New Summaries

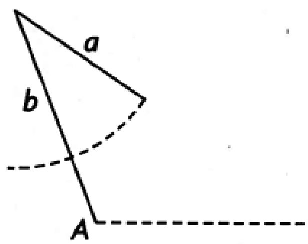
The ambiguous case arises in a SSA (side, side, angle) triangle. In this situation, depending on the size of the given angle and the lengths of the given sides, the sine law calculation may lead to 0, 1, or 2 solutions.

Need to Know

- In the ambiguous case, if $\angle A$, a, and b are given and $\angle A$ is acute, there are four cases to consider. In each case, the height of the triangle is $h = b \sin A$.

If $\angle A$ is acute and $a < h$, no triangle exists. 	If $\angle A$ is acute and $a = h$, one right triangle exists. 
If $\angle A$ is acute and $a > b$, one triangle exists. 	If $\angle A$ is acute and $h < a < b$, two triangles exist. 

If $\angle A$, a, and b are given and $\angle A$ is obtuse, there are two cases to consider.

If $\angle A$ is obtuse and $a < b$ or $a = b$, no triangle exists. 	If $\angle A$ is obtuse and $a > b$, one triangle exists.