Date: ____________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) prove trigonometric identities.

Last day's work: pp. 338-339 #1 – 5, 8 – 13
p. 340 #2

338 2, 8, 12
13
3C

We will discuss p.339 #13 at the start of class tomorrow.

p. 338 2, 3c 8

2c) $\tan 30° + 2(\sin 45°)(\cos 60°)$

$$= \frac{1}{\sqrt{3}} + 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

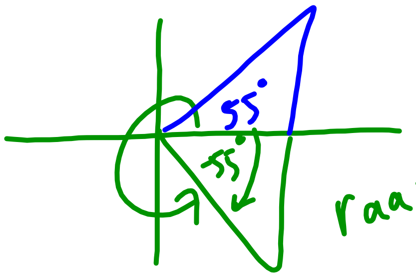
$$= \frac{\sqrt{3}}{3} + \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{3} + \frac{\sqrt{2}}{2}$$

$$= \frac{2\sqrt{3}}{6} + \frac{3\sqrt{2}}{6}$$

$$= \frac{2\sqrt{3}+3\sqrt{2}}{6}$$

3c) $\cos(-55°)$



raa = 55°

$\theta = 305°$

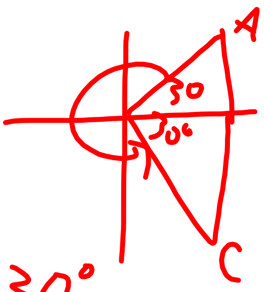
or $\theta = 55°$

p. 340 #2

$0 \le \theta \le 360$

2b) $\cos\theta = \frac{\sqrt{3}}{2}$

raa: $\theta = 30$



$\theta = 30°$

or

$\theta = 330°$

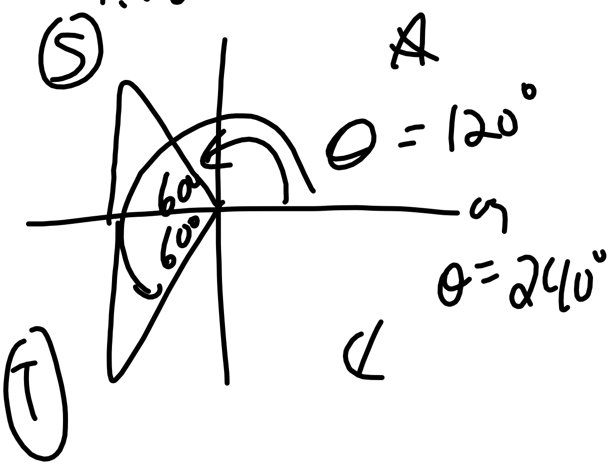
2d) $\sec\theta = -2$

$\cos\theta = \frac{-1}{2}$

if $\cos\theta = \frac{1}{2}$

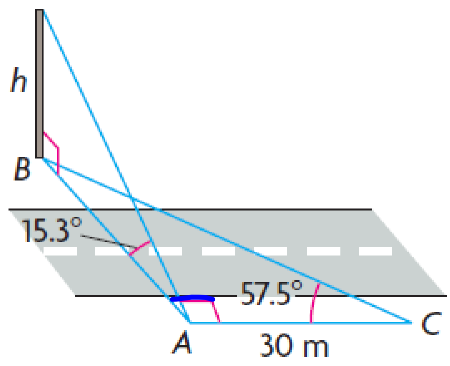
$\theta = 60°$

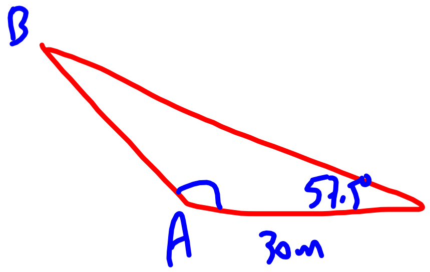
$\therefore$ raa $= 60°$

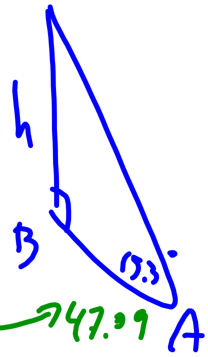


p. 339

12. To determine the height of a pole across a road, Justin takes two measurements. He stands at point *A* directly across from the base of the pole and determines that the angle of elevation to the top of the pole is 15.3°. He then walks 30 m parallel to the freeway to point *C*, where he sees that the base of the pole and point *A* are 57.5° apart. From point *A*, the base of the pole and point *C* are 90.0° apart. Calculate the height of the pole to the nearest metre.







$\tan 57.5° = \frac{BA}{30}$

$BA = 30 \tan 57.5°$

$\doteq 47.09$

$\tan 15.3° \doteq \frac{h}{47.09}$

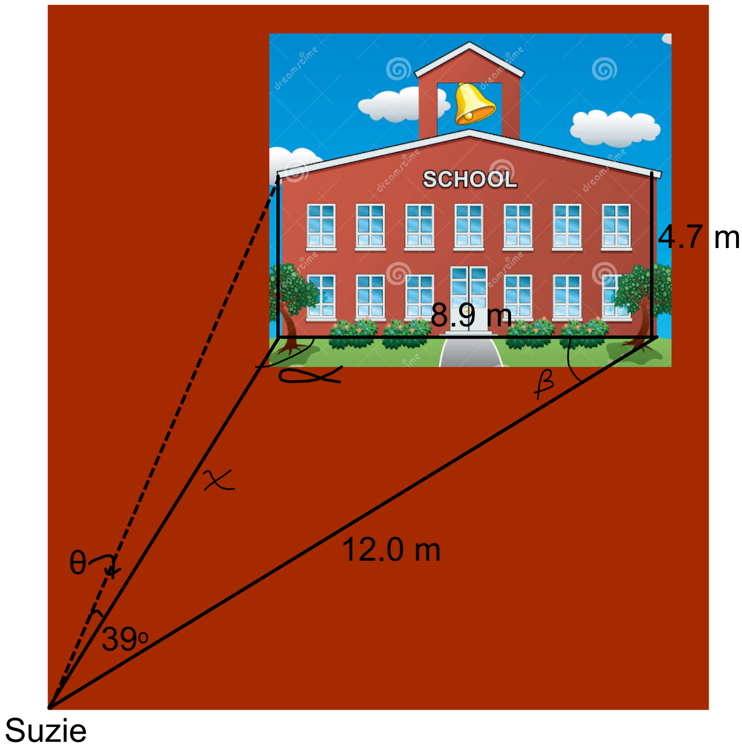
$h \doteq 47.09 \tan 15.3°$

$\doteq 12.8$

$\doteq 13$ m

p. 339 **Soluon #1**

13. While standing at the left corner of the schoolyard in front of her school, Suzie estimates that the front face is 8.9 m wide and 4.7 m high. From her position, Suzie is 12.0 m from the base of the right exterior wall. She determines that the left and right exterior walls appear to be 39° apart. From her position, what is the angle of elevation, to the nearest degree, to the top of the left exterior wall?



$$\frac{\sin\alpha}{12} = \frac{\sin 39^\circ}{8.9}$$

$$\alpha = \sin^{-1}\left(12 \times \frac{\sin 39^\circ}{8.9}\right)$$

$$\doteq 58.05^\circ$$

But α is obtuse

$$\therefore \alpha \doteq 121.9^\circ$$

$$\therefore \beta \doteq 180^\circ - 39^\circ - 121.9^\circ$$

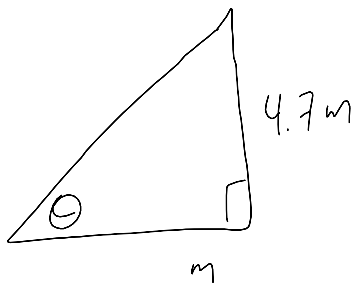
$$\doteq 19.1^\circ$$

$$\frac{x}{\sin 19.1^\circ} = \frac{8.9}{\sin 39^\circ}$$

$$x \doteq \sin 121.9^\circ \times \frac{8.9}{\sin 39^\circ}$$

$$\doteq 4.62$$

$$\doteq 4.6$$



$$\tan\theta \doteq \frac{4.7}{4.6}$$

$$\theta \doteq \tan^{-1}\left(\frac{4.7}{4.6}\right)$$

$$\doteq 45.6$$

$$\doteq 46^\circ$$

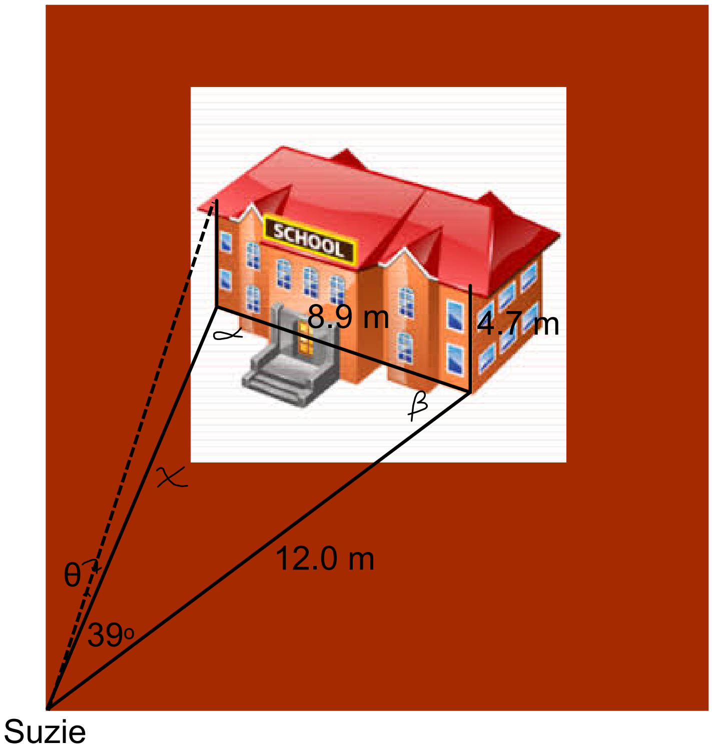
$\therefore$ the angle of elevation is 46°

[some texts have 46° at back.]

but see next page.

p. 339 **Soluon #2**

13. While standing at the left corner of the schoolyard in front of her school, Suzie estimates that the front face is 8.9 m wide and 4.7 m high. From her position, Suzie is 12.0 m from the base of the right exterior wall. She determines that the left and right exterior walls appear to be 39° apart. From her position, what is the angle of elevation, to the nearest degree, to the top of the left exterior wall?



$$\frac{\sin\alpha}{12} = \frac{\sin 39^\circ}{8.9}$$

$$\alpha = \sin^{-1}\left(12 \times \frac{\sin 39^\circ}{8.9}\right)$$

$$\doteq 58.05^\circ$$

$$\therefore \beta \doteq 180^\circ - 39^\circ - 58.1^\circ$$

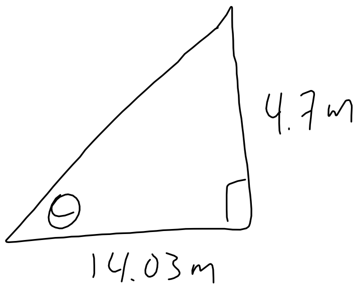
$$\doteq 82.9^\circ$$

$$\frac{x}{\sin 82.9^\circ} \doteq \frac{8.9}{\sin 39^\circ}$$

$$x \doteq \sin 82.9^\circ \times \frac{8.9}{\sin 39^\circ}$$

$$\doteq 14.033$$

$$\doteq 14.03$$



$$\tan\theta \doteq \frac{4.7}{14.03}$$

$$\theta \doteq \tan^{-1}\left(\frac{4.7}{14.03}\right)$$

$$\doteq 18.51$$

$$\doteq 19^\circ$$

$\therefore$ the angle of elevation is 19°.

Some texts say 18° at the back?

5.5 Trigonometric Identities

Date: May 8/17

Equations are valid for only certain values of the variable.
For example:

$2x + 1 = 7$

$2x = 7 - 1$

$2x = 6$

$x = 3$

$x = 3$ is the only value to make the equation true.

$x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$

$\therefore x = 7$ or $x = -2$

$x = 7$ and $x = -2$ are the only values to make the equation true.

Identities are valid for **all values** of the variable.
For example:

$2(x + 3) = 2x + 6$ $\qquad$ $x^2 + 6x + 9 = (x + 3)^2$

Let's start with the circle definitions to develop some identities that we can use later.

SYR CXR TYX

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

To Prove an Identity:

*** Separate the LS and RS, and work on them separately**

Ex.1 Prove that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Restriction

$\cos\theta \neq 0$

$\theta \neq \cos^{-1}(0)$

$\theta \neq 90°$

$LS = \tan\theta$

$= \dfrac{y}{x}$

$RS = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\frac{y}{r}}{\frac{x}{r}}$

$= \dfrac{y}{r} \div \dfrac{x}{r}$

$= \dfrac{y}{\cancel{r}} \times \dfrac{\cancel{r}}{x}$

$= \dfrac{y}{x}$

$\because LS = RS$

$\therefore$ QED

Q.E.D. (also written QED)

"quod erat demonstrandum"

"that which was to be demonstrated"

Ex.2 Prove that $\sin^2\theta + \cos^2\theta = 1$

$LS = \sin^2\theta + \cos^2\theta$ $RS = 1$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$*$ But $x^2 + y^2 = r^2$ (PT)

$= \frac{y^2 + x^2}{r^2}$

$\therefore LS = RS$

$= \frac{r^2}{r^2}$

$\therefore$ QED

$= 1$

Ex.3 Prove that Use "known" identities; i.e. known since Ex.1&2

a) $\dfrac{\cos\alpha\tan\alpha}{\sin\alpha}=1$

b) $\cos\phi=\dfrac{1}{\cos\phi}-\sin\phi\tan\phi$

$LS=\dfrac{\cos\alpha\tan\alpha}{\sin\alpha}$ $RS=1$

$=\dfrac{\cancel{\cos\alpha}\left(\dfrac{\sin\alpha}{\cancel{\cos\alpha}}\right)}{\sin\alpha}$

$=\dfrac{\sin\alpha}{\sin\alpha}$

$=1$

$\therefore LS=RS$

$\therefore$ QED.

$LS=\cos\phi$ $RS=\dfrac{1}{\cos\phi}-\sin\phi\tan\phi$

$=\dfrac{1}{\cos\phi}-\sin\phi\left(\dfrac{\sin\phi}{\cos\phi}\right)$

$=\dfrac{1}{\cos\phi}-\dfrac{\sin^2\phi}{\cos\phi}$

$\because \sin^2\phi+\cos^2\phi=1$ $=\dfrac{1-\sin^2\phi}{\cos\phi}$

$=\dfrac{\cos^2\phi}{\cos\phi}$

$=\dfrac{(\cos\phi)\cancel{(\cos\phi)}}{\cancel{\cos\phi}}$

$=\cos\phi$

$\therefore LS=RS$

$\therefore$ QED

Identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 338-339 #1 – 5, 8 – 13
p. 340 #2

Study for the Unit 5 Summative!

Today's Homework Practice includes:
p. 310 #1 – 6

Work ahead? pp. 310-311 #8, 10 – 12 [14]
Worksheet a – j (***online***)

Note: Sometimes using substitution can help simplify a question.

Ex. Simplify $(1 - \cos\theta)(1 + \cos\theta)$ Change to $(1 - a)(1 + a)$

$= 1 - \cos^2\theta$

$= 1 + a - a - a^2$

$= 1 - a^2$