

Before we begin, are there any questions from last day's work?

p. 390 #1 and 2

p. 391 #4 (**in your table of values choose  $x$  from -3 to 3. Also, look in the text's answer section to check your graphs instead of a graphing calculator.**)

p. 391 #6

## Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) use an exponential relation to model exponential growth or decay
- b) solve problems involving exponential growth or decay.

MBF 3CI

Date: May 17/17

### 7.5 Modelling Exponential Growth and Decay Problems

Population growth and price increases can be modeled by increasing exponential graphs. Decreasing exponential relations can represent declining populations and depreciation in value.

An **exponential relation** is of the form  $y = a(b^x)$ , where  $a$  and  $b$  are numbers, and  $x$  and  $y$  are variables. Also,  $b > 0$  and  $b \neq 1$ .

where  $a$  is the original (starting) value, also the y-intercept  
 $b$  is the base; the *growth factor* or *decay factor*  
 $x$  is the time, or number of years

For decay:  $0 < b < 1$

For growth:  $b > 1$

$b$



**Exponential Growth**

Ex.1 In 2001, the population of Canada was 31 051 000.

The annual growth rate is assumed to be about 1%.

a) What is the change factor as a decimal?

Recall:  $y = a(b)^x$

Next year's population is 100 % + 1 %

(this year's) (growth)

= 101 %

the base,  $b = 1.01$  (as a decimal)... "b" is the "growth factor" or "change factor"

b) Write a formula that models the population  $y$ ,  
with the number of years since 2001  $x$ .

$$y = 31\,051\,000(1.01)^x$$

c) Predict the population of Canada in 2010, to the nearest thousand.

$$y = 31\,051\,000(1.01)^9$$

$$\approx 33\,960\,021.4$$

$$\approx 33\,960\,000$$

$\therefore$  the population in 2010 should be 33 960 000

2010 - 34,149,200 (Est.)

Ex.2 A population increases by 7.8% each year.

If the initial population is 124, determine the population after:

a) 5 years

b) 10 years

Let  $y$  represent the population.

Let  $x$  represent the number of years.

$$a = 124$$

$$a = 124$$

$$b = 100\% + 7.8\%$$

$$b = 1.078$$

$$b = 107.8\%$$

$$b = 1.078$$

$$y = 124(1.078)^x$$

$$y = 124(1.078)^5$$

$$y = 124(1.078)^{10}$$

$$y \approx 180.5$$

$$y \approx 262.7$$

$$y \approx 180.5$$

$\therefore$  after 10 years the population is 262

$\therefore$  after 5 years the population is 180.

$$y = a(b^x)$$

**Exponential Decay**

Ex. 1 The price of a new car is \$24 599. Its value depreciates by 30% each year.

a) Write a formula to model the decrease in value of the new car.

DEFINE YOUR VARIABLES.

👉 Let  $y$  represent the value of the car, in dollars. 👉 Recall:  $y = a(b)^x$

👉 Let  $x$  represent the number of years. 👉 The base,  $b$ , must represent a 30% decrease:

👉  $b = 100\% - 30\%$  👉

$= 70\%$

$= 0.70$

$\therefore y = 24\,599(0.70)^x$  👉

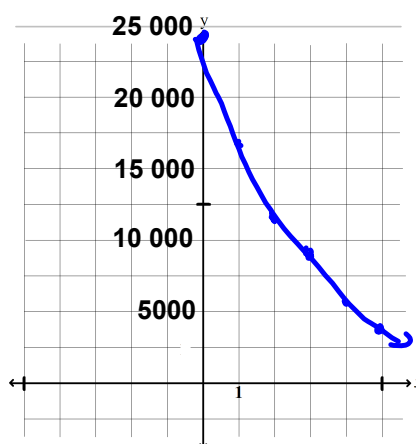
b) What is the depreciated value of the car after 5 years?

👉  $y = 24\,599(0.7)^5$

👉  $\div \$4\,134.35$

c) Graph this relation that models that value of the car for the first five years.

$x$	$y$
0	24,599
1	17,219.30
2	12,053.51
3	8,437.46
4	5,906.22
5	4,134.35



Ex.2 Caffeine is present in coffee, tea, chocolate and other products.

This chemical is eliminated from the human body over time exponentially.

The table shows the mass of caffeine remaining in an average-sized person drinking a cup of coffee containing 130 mg of caffeine:

<b>time</b>	0	1	2	3	4	5	6	7
<b>mg</b>	130	113.1	98.4	85.6	74.5	64.8	56.4	49.0

a) Why is it an example of exponential decay?

👉 Check the y-ratios.

$$\begin{array}{ccc} \frac{113.1}{130} & \frac{98.4}{113.1} & \frac{85.6}{98.4} \\ =0.87 & =0.870 & =0.869 \end{array}$$

the y-ratios are constant,  $\therefore$  the table represents exponential decay

👉 we know it is decay, because the base is between 0 and 1 👉

b) What percent of caffeine is eliminated from a person's body per hour?

👉  $b=0.87$

👉  $\therefore$  approximately 87% is **remaining**. (the base represents the % remaining)

👉  $\therefore$  **13% of the caffeine is being eliminated from a person's body per hour.**

Entertainmentp. 401 #1bcd, 2bc, 5abcd, 6, 7ab, 12

Return and correct Show What You Know 7.1? (next page)

**Be ready for SWYK 7.2 !**

**(It will have more exponent law questions, plus some sketching.)**

Exponent Law Practice: Simplify w/ pos. exp.

$$a) (5^{10})^2$$

$$= 5^{10 \times 2}$$

$$= 5^{20}$$

$$b) 4^{12} \div 4^2$$

$$= 4^{12-2}$$

$$= 4^{10}$$

$$c) 3^4 \times 3^5$$

$$= 3^{4+5}$$

$$= 3^9$$

$$d) 2^6 \div 2^9$$

$$= 2^{6-9}$$

$$= 2^{-3}$$

$$= \left(\frac{1}{2}\right)^3$$

$$e) \left( (13^4)^2 \times (13^6) \right)^0 \quad f) \left( \frac{3}{5} \right)^{-2}$$

$$= 1$$

$$= \left( \frac{5}{3} \right)^2$$

$$= \frac{5^2}{3^2}$$

$$= \frac{25}{9}$$