Date: ____________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

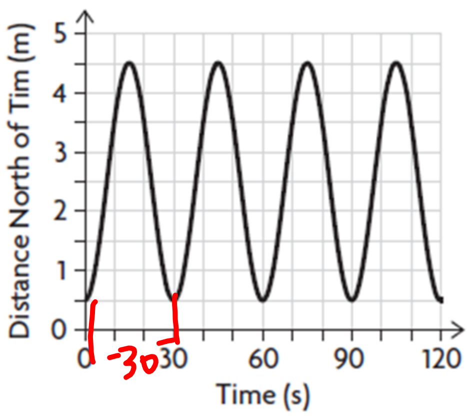
a) solve problems related to real-world applications of sinusoidal functions.

Last day's work: pp. 391-393 #1 – 6, 9, 12 [13,14]

6.7 Solving Problems Using Sinusoidal Models

Date: May 24/17

Ex. 1 Tim has a model train that goes around a circular train track,
and Tim is standing directly south of the track.
The graph below shows the train's distance north of Tim as a function of time.



$$y = \frac{max+min}{2} = \frac{4.5+0.5}{2} = \frac{5}{2} = 2.5$$

$$a = max - c = 4.5 - 2.5 = 2$$

$$a = \frac{max - min}{2} = \frac{4.5-0.5}{2} = \frac{4}{2} = 2$$

a) What is the equation of the axis of the function? $y = 2.5$

b) What is the amplitude of the function, and what does it represent in this situation?? a = 2; the radius of the track

c) What is the period of the function, and what does it represent in this situation?? 30 s; time for 1 lap around the track

d) What is the range of the function? $\{y \quad \mathbf{R}/0.5 \le y \le 4.5\}$

e) Determine the equation of the sinusoidal function. $y = -2\cos(12x) + 2.5$

f) What is the train's distance north of Tim at $t = 52$ s?

Sub $t = 52$ s in equation above, then $y = 2.709$ m

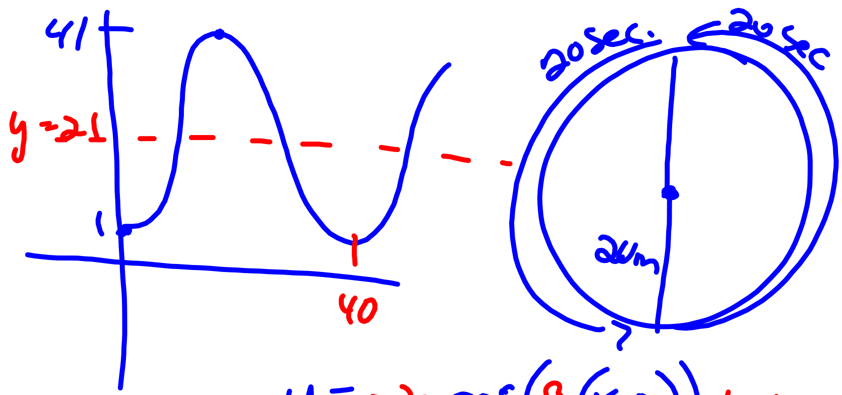
$$k = \frac{360}{period} = \frac{360}{30} = 12$$

Ex. 2

A Ferris wheel with radius 20 metres rotates once every 40 seconds.
Passengers get on at the bottom of the wheel, which is 1 metre off the ground.
Suppose you get on, and the wheel starts to rotate.

a) Write a sinusoidal equation which expresses your height as a function of elapsed time.

$h(t) = -20\cos(9t) + 21$



$y = -20\cos(9(x-0)) + 21$

$k = \frac{360}{40} = 9$

b) Calculate your height after 15 seconds.

35.14 m

Sub 15 for x in your eq'n.

c) If you are on the Ferris wheel for 5 minutes, how many complete rotations will you have completed?

5 min = 300 sec.
×60

$\frac{300 \text{ sec}}{40 \text{ sec}} = 7.5$

(7.5)
7 complete rotations

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 391-393 #1 – 6, 9, 12 [13,14]

4b 9be

5b 6 12

Today's Homework Practice includes:
pp. 398-401 #1 – 4, 6, 7, 9 [13]

Tomorrow's Review:
pp. 404-405 #1 – 3, 6, 8 – 10, 12, 13

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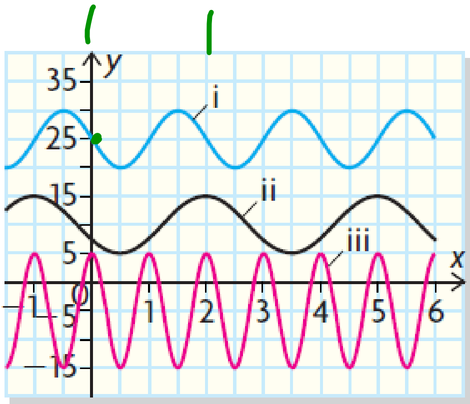
4. Determine the equation for each sinusoidal function.

b)



i) $d=0, \therefore a=-5$

period $=2$

$\therefore k = \frac{360}{2}$

$=180$

$c=25$

$y = -5\sin(180x)+25$

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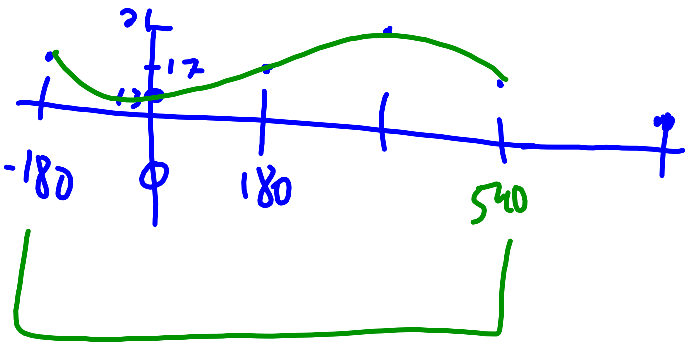
5. For each table of data, determine the equation of the function that is the simplest model.

b)

x	−180°	0°	180°	360°	540°	720°	900°
y	17	13	17	21	17	13	17



period $=720$

$c=17$

cos

$a = \frac{21-13}{2}$

$= \frac{8}{2}$

$=4$

$d=0$

$\therefore a=-4$

$k = \frac{360}{720}$

$= \frac{1}{2}$

$\therefore y = -4\cos\left(\frac{1}{2}x\right)+17$

$y = 4\cos\frac{1}{2}(x-180)+17$

NOT CORRECT

But $y = 4\cos\left(\frac{1}{2}x-180\right)+17$

$= 4\cos\left(\frac{1}{2}(x-360)\right)+17$

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6. Determine the equation of the cosine function whose graph has each of the following features.

	Amplitude	Period	Equation of the Axis	Horizontal Translation
a)	3	360°	$y = 11$	0°
b)	4	180°	$y = 15$	30°
c)	2	40°	$y = 0$	7°
d)	0.5	720°	$y = -3$	−56°

6. a) Since the amplitude is 3, a in the function $y = a\cos(k(x - d)) + c$ is 3. Since the period is 360°, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{360}$ or 1. Since the equation of the axis is $y = 11$, c in the equation $y = a\cos(k(x - d)) + c$ is 11. Since the horizontal translation is 0°, d in the equation $y = a\cos(k(x - d))$ is 0. Therefore, the equation is $y = 3\cos x° + 11$. This can also be expressed as is $y = 3\sin(x + 90)° + 11$.

b) Since the amplitude is 4, a in the function $y = a\cos(k(x - d)) + c$ is 4. Since the period is 180°, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{180}$ or 2. Since the equation of the axis is $y = 15$, c in the equation $y = a\cos(k(x - d)) + c$ is 15. Since the horizontal translation is 30°, d in the equation $y = a\cos(k(x - d))$ is 30. Therefore, the equation is $y = 4\cos[2(x - 30)]° + 15$. This can also be expressed as $y = 4\sin[2(x + 15)]° + 15$.

c) Since the amplitude is 2, a in the function $y = a\cos(k(x - d)) + c$ is 2. Since the period is 40°, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{40}$ or 9. Since the equation of the axis is $y = 0$, c in the equation $y = a\cos(k(x - d)) + c$ is 0. Since the horizontal translation is 7°, d in the equation $y = a\cos(k(x - d))$ is 7. Therefore, the equation is $y = 2\cos 9(x - 7)°$. This can also be expressed as is $y = 2\sin 9(x + 3)°$.

d) Since the amplitude is 0.5, a in the function $y = a\cos(k(x - d)) + c$ is 0.5. Since the period is 720°, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{720}$ or $\frac{1}{2}$. Since the equation of the axis is $y = -3$, c in the equation $y = a\cos(k(x - d)) + c$ is −3. Since the horizontal translation is −56°, d in the equation $y = a\cos(k(x - d))$ is −56. Therefore, the equation is $y = 0.5\cos\left[\frac{1}{2}(x + 56)\right]° - 3$.

This can also be expressed as

$y = 0.5\sin\left[\frac{1}{2}(x + 236)\right]° - 3$.

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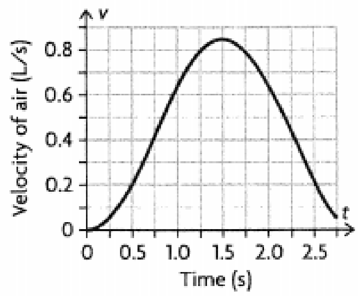
9. The table shows the velocity of air of Nicole's breathing while she is at rest.

Time (s)	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2	2.25	2.5	2.75	3
Velocity (L/s)	0	0.22	0.45	0.61	0.75	0.82	0.85	0.83	0.74	0.61	0.43	0.23	0

a) Explain why breathing is an example of a periodic function.
b) Graph the data, and determine an equation that models the situation.
c) Using a graphing calculator, graph the data as a scatter plot. Enter your equation and graph. Comment on the closeness of fit between the scatter plot and the graph.
d) What is the velocity of Nicole's breathing at 6 s? Justify.
e) How many seconds have passed when the velocity is 0.5 L/s?

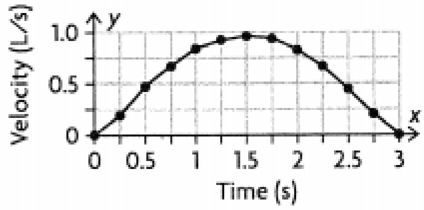
9. a) The respiratory cycle is an example of a periodic function because we inhale, rest, exhale, rest, inhale, and so on in a cyclical pattern.

b)



The amplitude is half the distance between the maximum and minimum values. Since the minimum is 0 and the maximum is 0.85, the amplitude is $\frac{0.85 - 0}{2}$ or 0.425. Since the amplitude is 0.425, a in the equation $v = a\cos(k(t - d)) + c$ is 0.425. The period is the change in t that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $t = 0$ and $t = 3$, the period is $3 - 0$ or 3. Since the period is 3, k in the equation $v = a\cos(k(x - d)) + c$ is $\frac{360}{3}$ or 120. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 0 and the maximum is 0.85, the equation of the axis is $y = \frac{0.85 + 0}{2}$ or $y = 0.425$. Since the equation of the axis is $y = 0.425$, c in the equation $v = a\cos(k(t - d)) + c$ is 0.425. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Since there is a reflection in the x-axis, the sign of a in the equation $v = a\cos(k(t - d))$ should be negative. Since there is no horizontal translation, d in the equation $v = a\cos(k(t - d))$ is 0. Therefore, the equation is $v = -0.425\cos(120t)^\circ + 0.425$. This can also be expressed as $v = -0.425\sin(120t + 90)^\circ + 0.425$.

c)



The equation is almost an exact fit on the scatter plot.

d) The first cycle goes from 0 s to 3 s. Therefore, the second cycle goes from 3 s to 6 s. Since each cycle is the same, the velocity of Nicole's breathing at 3 s is the same as the velocity of Nicole's breathing at 6 s. Therefore, the velocity of Nicole's breathing at 6 s is 0 L/s.

e) Since the equation for this situation is

$$v = -0.425\cos(120t)^\circ + 0.425,$$
$$0.5 = -0.425\cos(120t)^\circ + 0.425$$
$$0.075 = -0.425\cos(120t)^\circ$$
$$-0.176 = \cos(120t)^\circ$$
$$100.164 = 120t$$
$$t = 0.8\text{ s}$$

Since 0.8 s have passed when the velocity is 0.5 L/s, and since the maximum occurs at 1.5 s, the difference between the time at the maximum and the time at velocity 0.5 L/s is 1.5 s − 0.8 s or 0.7 s. Since the curve is symmetrical, a velocity of 0.5 L/s also occurs at 1.5 s + 0.7 s or 2.2 s.

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12. Describe a procedure for writing the equation of a sinusoidal function based on a given graph.

12. Find the amplitude. Whatever the amplitude is, a in the equation $y = a\cos(k(x - d)) + c$ will be equal to it. Find the period. Whatever the period is, k in the equation $y = a\cos(k(x - d)) + c$ will be equal to 360 divided by it. Find the equation of the axis. Whatever the equation of the axis is, c in the equation $y = a\cos(k(x - d)) + c$ will be equal to it. Find the phase shift. Whatever the phase shift is, d in the equation $y = a\cos(k(x - d)) + c$ will be equal to it. Determine if the function is reflected in its axis. If it is, the sign of a will be negative; otherwise, it will be positive. Determine if the function is reflected in the y-axis. If it is, the sign of k will be negative; otherwise, it will be positive.