Before we begin, are there any questions from last day's work?

Today's Learning Goal(s):

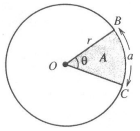
By the end of the class, I will be able to:

a) calculate the arc length of circle.
b) calculate the area of a sector of a circle.

6.9.1: Arc Length and Sector Area

Date: May 30/17

There are problems that often occur in industry that involve arcs and sectors of circles.
Consider the following diagram:



A circle of radius r is drawn, with sector *BOC* bounded by 2 radii, *OB* and *OC*, and an arc *BC*, of length *a*.
The area of the sector is A,
and the sector angle at the centre *O* is θ, measured in degrees.

We can use the proportional relationship: $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{sector area}}{\text{area of circle}} = \frac{\text{sector angle}}{\text{complete rotation}}$

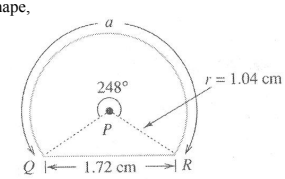
So, if $\frac{a}{2\pi r} = \frac{A}{\pi r^2} = \frac{\theta}{360°}$

then $\frac{a}{2\pi r} = \frac{\theta}{360°}$ and $\frac{A}{\pi r^2} = \frac{\theta}{360°}$

and isolating, [arc length] $a = 2\pi r\left(\frac{\theta}{360°}\right)$ and [sector area] $A = \pi r^2\left(\frac{\theta}{360°}\right)$

Thus both the arc length, *a*, and sector area, *A*, can be calculated
once the radius, *r*, and the sector angle, θ, in degrees, are known.

Ex. 1 A cam for a sewing machine's stitching-control cycle is circular in shape, with a flat side, and has the dimensions shown.
a) Calculate the total perimeter of the cam. (to 3 decimal places)



$P = \overset{\frown}{} + \_\_\_$

$= \frac{\theta}{360}(2\pi r) + 1.72$

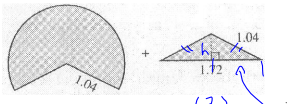
$= \frac{248}{360}(2\pi(1.04)) + 1.72$

$\doteq 4.5015 + 1.72$

$\doteq 4.502 + 1.72$

$\doteq 6.222$ cm

b) If the cam is 0.36 cm thick and is made from an alloy whose density is 3.8g/cm^3, determine the mass of the cam. (to 3 decimal places)
Hint: the cam is a prism, Volume $_{\text{prism}}$ = Area $_{\text{base}}$ x height, then mass = volume x density



$\frac{1.72}{2} = 0.86$

A_{base} = Sector Area + Triangle

$= \frac{\theta}{360}\pi r^2 + \frac{1}{2}bh$

$\doteq \frac{248}{360}\pi(1.04)^2 + \frac{1}{2}(1.72)(0.585)$

$\doteq 2.340 + 0.499$

$\doteq 2.8439$

$\doteq 2.844$ cm^2

$h^2 = 1.04^2 - 0.86^2$

$h = \sqrt{0.342}$

$\doteq 0.5848$

$\doteq 0.585$

From above $V = A_{base} \times h$

$\doteq 2.844 \times 0.36$

$\doteq 1.0238$

$\doteq 1.024$ cm^3

Mass = Density x Volume

$\doteq 3.8\frac{g}{cm^3} \times 1.024 cm^3$

$\doteq 3.8912$

$\doteq 3.891$ g

∴ the mass of the cam is 3.891 g.