Date: _____

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) calculate the sum of the terms of an arithmetic series.

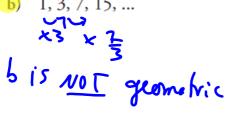Last day's work: pp. 430-432 #1 − 3, 5 − 11 [18-20]

10

56

8

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5. i) Determine whether each sequence is geometric.

   ii) If a sequence is geometric, state the general term and the recursive formula.

   a) 12, 24, 48, 96, ...   **d)** 5, −15, 45, −135, ...

   b) 1, 3, 7, 15, ...   **e)** 6, 7, 14, 15, ...

$\times 3 \quad \times \frac{7}{3}$

b is NOT geometric

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8. Determine the recursive formula and the general term for the geometric sequence in which

   a) the first term is 19 and the common ratio is 5

   b) $t_1 = -9$ and $r = -4$

   c) the first term is 144 and the second term is 36

   d) $t_1 = 900$ and $r = \dfrac{1}{6}$

d) g. seq.

$$a = 900$$

$$r = \frac{1}{6}$$

$$t_n = ar^{n-1}$$

$$\therefore t_1, t_2, t_3, \ldots$$

$$900, 150, 25, \ldots$$

$$t_n = \frac{1}{6} t_{n-1}$$

$$t_n = 900\left(\tfrac{1}{6}\right)^{n-1}$$

$$t_1 = 900$$

$$t_2 = 900\left(\tfrac{1}{6}\right)^{2-1}$$
$$= 900\left(\tfrac{1}{6}\right)$$
$$= 150$$

$$t_3 = 900\left(\tfrac{1}{6}\right)^{3-1}$$
$$= 900\left(\tfrac{1}{6}\right)^{2}$$
$$= 900\left(\tfrac{1}{36}\right)$$
$$= 25$$

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10. i) Determine whether each general term defines a geometric sequence, where $n \in \mathbf{N}$.

ii) If the sequence is geometric, state the first five terms and the common ratio.

a) $t_n = 4^n$

b) $t_n = 3^n + 5$

c) $f(n) = n^2 - 13n + 8$

d) $t_n = 7 \times (-5)^{n-4}$

e) $f(n) = \dfrac{2}{3n+1}$

f) $f(n) = \dfrac{11}{13^n}$

$t_1 = \dfrac{11}{13}$

$t_2 = \dfrac{11}{13}\left(\dfrac{1}{13}\right)$

$t_3 = \dfrac{11}{13}\left(\dfrac{1}{13}\right)^2$

$t_n = \dfrac{11}{13}\left(\dfrac{1}{13}\right)^{n-1}$

$\therefore a = \dfrac{11}{13} \quad r = \dfrac{1}{13}$

$t_1 = \dfrac{11}{13} \quad , \quad t_2 = \dfrac{11}{13^2} \quad t_3 = \dfrac{11}{13^3}$

$t_1 = 11\left(\dfrac{1}{13}\right) \qquad = \dfrac{11}{169} \qquad = \dfrac{11}{2197}$

try again

$t_2 = 11\left(\dfrac{1}{13}\right)^2 \quad t_3 = 11\left(\dfrac{1}{13}\right)^3$

$t_n = 11\left(\dfrac{1}{13}\right)^n$

$\therefore t_4 = \dfrac{11}{13^4} \qquad t_5 = \dfrac{11}{13^5}$

$\qquad = \dfrac{11}{28561} \qquad = \dfrac{11}{371293}$

7.5 Arithmetic Series

Date: *May 31/17*

Recall: An arithmetic sequence is a list of numbers with a common difference.

 An **Arithmetic Series** is the **sum** of the terms of an arithmetic sequence.

    Sequence: 3, 5, 7, 9, ...
      Series: 3 + 5 + 7 + 9 + ...

Karl Friedrich Gauss
(1777-1855)

There's a famous (and probably apocryphal) story about the mathematician
Karl Friedrich Gauss that goes something like this:

apocryphal (of a story or statement) of doubtful authenticity, although widely circulated as being true.

Gauss was 9 years old, and sitting in his math class.
He was a genius even at this young age, and as such was incredibly bored in his class
and would always goof off and get into trouble.
One day his teacher, J.G. Büttner, wanted to punish him for goofing off, and told him
that if he was so smart, why didn't he go sit in the corner and add up all the integers from
1 to 100? Gauss went and sat in the corner, but didn't pick up his pencil.
The teacher confronted him, saying "Karl! Why aren't you working? I suppose you've
figured it out already, have you?"
Gauss responded with "Yes – it's 5,050." The teacher didn't believe him and spent the
next ten minutes or so adding everything up by hand, only to find that Gauss was right!

So how did Gauss find the answer so fast? What did he see that his teacher didn't?
The answer is simple, really – it's all about pattern recognition.
Let's look at the problem more closely.

Ex.1   Add the following:

$$S_n = 1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100$$

② ☞ $S_n = 100 + 99 + 98 + 97 + \ldots + 4 + 3 + 2 + 1$ ☜ ①

③ ☞

④ ☞ $2S_n = 101 + 101 + 101 + 101 + \ldots + 101 + 101 + 101 + 101$

⑤ ☞ $2S_n = 100\ (101)$

☞ $S_n = \dfrac{100(101)}{2}$

☞ $= 5050$

S_n -   partial sum ☞

　　　　 or the sum of the first n terms ☞

☞ Note: From Gauss, the sum of the first n numbers: $S_n = \dfrac{n(n+1)}{2}$

In general: $nd - 2d + d$
$nd - d$
$d(n-1)$

$$S_n = a \quad + \quad a+d \quad + \quad a+2d \quad + \ldots + \quad a+(n-3)d \; +a+(n-2)d \; +a+(n-1)d \overset{t_n}{\underset{①}{}}$$

$$S_n = a+(n-1)d + a+(n-2)d + a+(n-3)d + \quad \ldots \quad + a+2d \; + \; a+d \quad + \quad a$$

$$2S_n = 2a+(n-1)d+2a+(n-1)d+2a+(n-1)d+ \; \ldots \; 2a+(n-1)d + 2a+(n-1)d + 2a+(n-1)d$$

n terms

$$2S_n = n[2a+(n-1)d]$$

$$\frac{n}{2}\left[a + a + (n-1)d\right]$$

The Arithmetic Series Formula:

$$S_n = \frac{n}{2}[2a+(n-1)d] \quad \textbf{or} \quad S_n = \frac{n}{2}[t_1 + t_n]$$

where n is the term's position number,
$\qquad a = t_1$,
$\qquad d$ is the common difference, and
$\qquad t_n =$ last term

Ex.2   Find S_{50} for $8 + 11 + 14 + ... + t_n$

a series

$d = 3$

$a = 8$

$n = 50$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$t_n = a + (n-1)d$

$S_{50} = \frac{50}{2}[2(8) + 49(3)]$

$= 25[16 + 147]$

$= 25(163)$

$= 4075$

$S_{50} = 4075$

Ex.3   Find the sum of: $-12 - 7 - 2 + 3 + ... + 138$

a -series   Find n

$a = -12$

$d = 5$

$n = ?$

$t_n = 138$

$t_n = a + (n-1)d$

$S_n = \frac{n}{2}(a + t_n)$

$t_n = a + (n-1)d$

$138 = -12 + (n-1)(5)$

$138 = -12 + 5n - 5$

$138 = 5n - 17$

$138 + 17 = 5n$

$155 = 5n$

$\frac{155}{5} = n$

$31 = n$

$n = 31$

$S_{31} = 1953$

$S_{31} = \frac{31}{2}(-12 + 138)$

$= \frac{31}{2}(126)$

$= 1953$

$S_{31} = \frac{31}{2}[2(-12) + 30(5)]$

$= 1953$

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 430-432 #1 − 3, 5 − 11 [18-20]
p. 447 #1 − 6

Today's Homework Practice includes:
pp. 452-453 #(1 − 7)ace, 11, 13 [15,16]

Tomorrow: **SWYK 7.1** (on Sequences)