

Date: _____

Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) calculate the "present value" of an amount being charged or earning compound interest.

Last day's work: pp. 481-482 #5 – 10

pp. 490-492 #4 – 9, 11, 14 [20]

5,7,11

p. 491 #5

5. Sima invests some money in an account that earns a fixed rate of interest compounded annually. The amounts of the investment at the end of the first three years are shown at the right.

- Determine the annual rate of compound interest earned.
- How much did Sima invest?

Year	Total Amount
1	\$4240.00
2	\$4494.40
3	\$4764.06

No!

254.40

269.66

Use Ratios

$$\frac{4494.40}{4240} = 1.06$$

$$\frac{4764.06}{4494.40} = 1.06$$

b) Year 0 \rightarrow Principal Invested

$$\therefore 1 + 0.06 = r$$

$$\therefore i = 0.06$$

$$= 6\%$$

$$P = \frac{4240}{1.06}$$

$$= 4000$$

p. 491 #7

7. Serena wants to borrow \$15 000 and pay it back in 10 years. Interest rates are

A high, so the bank makes her two offers:

- Option 1: Borrow the money at 10%/a compounded quarterly for the full term.
- Option 2: Borrow the money at 12%/a compounded quarterly for 5 years and then renegotiate the loan based on the new balance for the last 5 years. If, in 5 years, the interest rate will be 6%/a compounded quarterly, how much will Serena save by choosing the second option?

$$A = P(1+i)^n$$

$$A = ?$$

$$P = 15\,000$$

$$i = \frac{0.10}{4}$$

$$n = 10 \times 4 = 40$$

$$A = 15\,000 \left(1 + \frac{0.10}{4}\right)^{40}$$

$$\approx 40\,275.957$$

$$\approx \$40\,275.96$$

$$A = P(1+i)^n$$

$$A = ?$$

$$P = 15\,000$$

$$i = \frac{0.12}{4}$$

$$n = 5 \times 4 = 20$$

$$A = 15\,000 \left(1 + \frac{0.12}{4}\right)^{20}$$

$$\approx 27\,091.668$$

$$\approx 27\,091.67$$

$$A = P(1+i)^n$$

$$A = ?$$

$$P = 27\,091.67$$

$$i = \frac{0.06}{4}$$

$$n = 5 \times 4 = 20$$

$$A = 27\,091.67 \left(1 + \frac{0.06}{4}\right)^{20}$$

$$\approx 36\,488.549$$

$$\approx \$36\,488.55$$

$$\therefore \text{Savings} = 40\,275.96 - 36\,488.55$$

$$\approx \$3\,787.41$$

p. 491 #11

11. Dieter deposits \$9000 into an account that pays 10%/a compounded quarterly. After three years, the interest rate changes to 9%/a compounded semi-annually. Calculate the value of his investment two years after this change.

$$11. A_3 = 9000 \left(1 + \frac{0.10}{4}\right)^{12}$$

$$= 12103.999$$

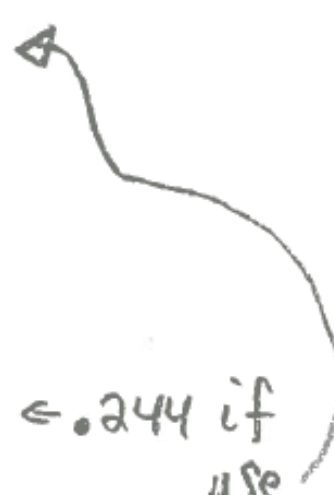
$$\approx \$12104$$

$$A_5 = 12104 \left(1 + \frac{0.09}{2}\right)^4$$

$$= 14434.245$$

$$\approx \$14434.25$$

* ← 0.244 if use



8.3 Compound Interest (Present Value)

From last day:

Date: _____

Simple Interest

$$A = P(1 + rt)$$

$$I = Prt$$

$$A = P + I$$

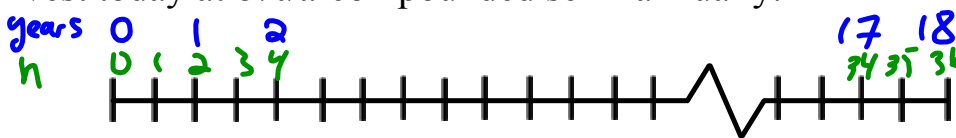
Compound Interest

$$A = P(1 + i)^n$$

i = interest rate per compounding period

n = number of compounding periods

Ex.1 Congratulations! You just had a baby girl. You want to to have \$30 000 for her education in 18 years. Determine the how much would you have to invest today at 8%/a compounded semi-annually.



$$A = 30000$$

$$P = ?$$

$$i = \frac{0.08}{2}$$

$$h = 18 \times 2 = 36$$

$$A = P(1 + i)^n$$

$$30000 = P\left(1 + \frac{0.08}{2}\right)^{36}$$

$$\frac{30000}{\left(1 + \frac{0.08}{2}\right)^{36}} = P$$

$$P = 7310.061$$

$$P = \$7310.06$$

\$7310.06

you would have to invest \$7310.06 today to have \$30 000 in 18 years.

Note: **Present value** is the principal that would have to be invested now to get a specific future amount (value) in a certain amount of time.

PV is used for present value instead of P, since P is used for principal.

Discuss: $A = P(1 + i)^n$ vs. $A = PV(1 + i)^n$ vs. $FV = PV(1 + i)^n$

$$PV = \frac{A}{(1 + i)^n}$$

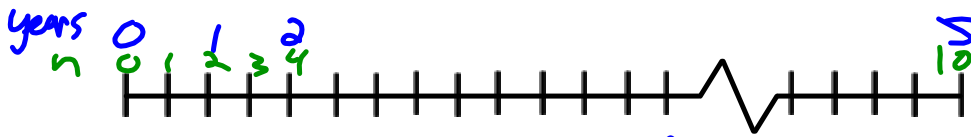
or $PV = A(1 + i)^{-n}$

$$= A\left(\frac{1}{(1 + i)^n}\right)$$

Ex.2

A local furniture company allows you to buy a living room set worth \$5000, if you agree to pay them \$8950 in 5 years.

Determine the annual interest rate, if interest will be compounded semi-annually.



$$A = 8950$$

$$A = P(1+i)^n$$

$$P = 5000$$

$$8950 = 5000\left(1 + \frac{i}{2}\right)^{10}$$

$$i = \frac{i}{2}$$

$$\frac{8950}{5000} = \left(1 + \frac{i}{2}\right)^{10}$$

$$h = 5 \times 2 = 10$$

$$\sqrt[10]{\frac{8950}{5000}} = 1 + \frac{i}{2}$$

$$\sqrt[10]{\frac{8950}{5000}} - 1 = \frac{i}{2}$$

$$2\left(\sqrt[10]{\frac{8950}{5000}} - 1\right) = i$$

$$i = 0.1198$$

$$= 11.98\%$$

12%

the annual interest rate being charged is 12%/a, compounded semi-annually.

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 481-482 #5 – 10

pp. 490-492 #4 – 9, 11, 14 [20]

Read the Key Ideas/Need to Know p.497

Today's Homework Practice includes:

pp. 498-499 #3 – 6, 8, 9, 11