Properties of Graphs of Functions (1.3)

Math Learning Target:

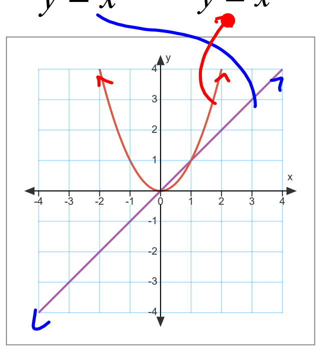
"I can compare properties between parent functions, and within a parent function's family."

A **transformation** is a geometric operation, such as a translation, reflection and compression.

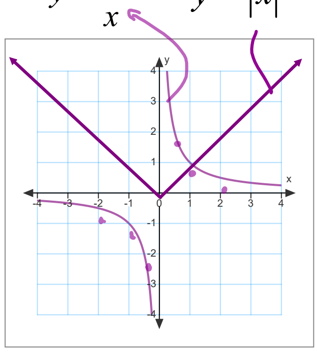
Each transformation is performed on a parent relation. There are very many parent relations. A **parent function** belongs to the set of parent relations and is the simplest function in a family of functions. For example, the family of quadratic functions are all constructed from $y = x^2$.

Here are the *seven* parent functions that will be used often...

$y = x$ $\quad$ $y = x^2$



$y = \frac{1}{x}$ $\quad$ $y = |x|$



$y = x$ Interval(s) of increase: $(-\infty,\infty)$
Interval(s) of decrease: none
End behaviours:
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$

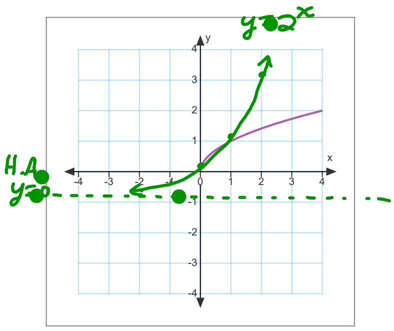
$y = \frac{1}{x}$ Interval(s) of increase: none
Interval(s) of decrease: $(-\infty,0)(0,\infty)$
End behaviours:
$x \to \infty, y \to 0^+$
$x \to -\infty, y \to 0^-$

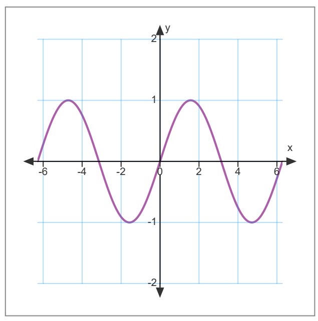
$y = x^2$ Interval(s) of increase: *$[0,\infty)$
Interval(s) of decrease: $(-\infty,0)$
End behaviours:
$x \to \infty, y \to \infty$
$x \to -\infty, y \to \infty$

$y = |x|$ Interval(s) of increase: *$[0,\infty)$
Interval(s) of decrease: $(-\infty,0)$
End behaviours:
$x \to \infty, y \to \infty$
$x \to -\infty, y \to \infty$

*include "0" in one interval or the other.

$y = \sqrt{x}$ $\quad$ $y = b^x$ i.e. $b = 2$ $\quad$ $y = \sin(x)$





$y = \sqrt{x}$ Interval(s) of increase: *$[0,\infty)$
Interval(s) of decrease: none
End behaviours:
$x \to \infty, y \to \infty$
~~$x \to 0, y \to 0$~~

$y = \sin(x)$ End behaviours:
Oscillating between -1 and 1

$y = 2^x$ Interval(s) of increase: $(-\infty,\infty)$
Interval(s) of decrease: none
End behaviours:
$x \to \infty, y \to \infty$
$x \to -\infty, y \to 0$

CHALLENGE! Can you determine expressions for the intervals of increase and intervals of decrease?

A function is **odd** when $f(-x) = -f(x)$

A function is **even** when $f(-x) = f(x)$

Example 1 Is $y = x^2$ even, odd, or neither? Prove algebraically.

$f(-x) = (-x)^2$ if $f(x) = x^2$

$= (-x)(-x)$ $\because f(-x) = f(x)$

$= x^2$ $\therefore$ even

$= f(x)$

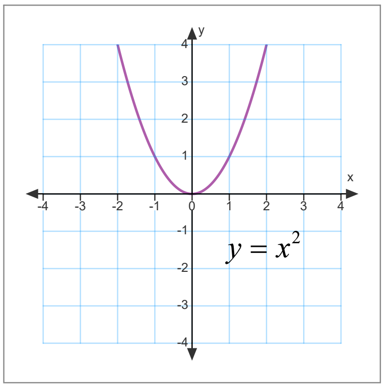
Example 2 Is $y = \frac{1}{x}$ even, odd, or neither? Prove algebraically.

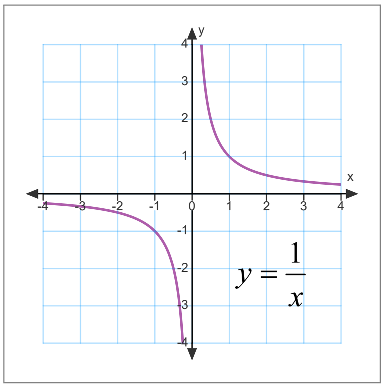
$f(x) = \frac{1}{x}$ $f(-x) = \frac{1}{(-x)}$ $-f(x) = -\left(\frac{1}{x}\right)$

$= \frac{1}{-x}$ $= \frac{-1}{x}$

$= -\frac{1}{x}$ $\because f(-x) = -f(x)$

$\therefore$ odd





Graphically, a function is even when

it is symmetric about the y-axis.

Graphically, a function is odd when

it has rotational symmetry about the origin

Do: pg. 23 #3*, 4ad, 5**, 6, 7, 8, 10***, 15

* Error in answer: the function can be derived from <u>any</u> $y=b^x$, for any valid "b"),

** The instructions are poor. Simply apply what was learned today in the lesson.

***In #10a, in the <u>instructions</u> for the question change $(-\infty, -2)$ to $(-\infty, 2]$

positive 2

square bracket

YES, you have permission to write in the textbook to make this change!