p.77 #10

10. A company that sells sweatshirts finds that the profit can be modelled by $P(s) = -0.30s^2 + 3.5s + 11.15$, where $P(s)$ is the profit, in thousands of dollars, and s is the number of sweatshirts sold (expressed in thousands).

a) Calculate the average rate of change in profit for the following intervals.

i) $1 \leq s \leq 2$ ii) $2 \leq s \leq 3$ iii) $3 \leq s \leq 4$ iv) $4 \leq s \leq 5$

b) As the number of sweatshirts sold increases, what do you notice about the average rate of change in profit on each sweatshirt? What does this mean?

c) Predict if the rate of change in profit will stay positive. Explain what this means.

i) $\frac{\Delta P}{\Delta n} = \frac{P(2) - P(1)}{2 - 1}$

$= \frac{-0.3(2)^2 + 3.5(2) + 11.15 - (-0.3(1)^2 + 3.5(1) + 11.15)}{2-1}$

$= 2.6$

$\therefore$ \$2.60 per shirt

ii) $\frac{\Delta P}{\Delta n} = \frac{P(3) - P(2)}{3 - 2}$

p.77 #10, 13

13. Vehicles lose value over time. A car is purchased for \$23 500, but is worth only \$8750 after eight years. What is the average annual rate of change in the value of the car, as a percent?

$$ARC = \frac{23500 - 8750}{0 - 8}$$

$$= \frac{14750}{-8}$$

$$= -\$1843.75/\text{yr.}$$

$$\% \text{ Change} = \frac{\text{change/diff.}}{\text{original}} \times 100\%$$

$$= \frac{-1843.75}{23500} \times 100$$

$$\doteq -7.84$$

$$\doteq -7.8\%$$

2.2 Estimating Instantaneous Rates of Change from T of V and Equations Part1-f17

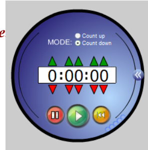
Estimating Instantaneous Rates of Change from Tables of Values and Equations (2.2) Part 1

Math Learning Target:

"I understand what average rate of change and instantaneous rate of change are, and the difference between the them.
Also, I can determine a reasonable estimate for the instantaneous rate of change using the methods presented, and interpret the result."

***INVESTIGATE** the Math (Page 79, A - G)*

10 min. time



Time, *t* (s)	6.0	6.2	6.4	6.6	6.8	7.0
Distance, *d*(*t*) (cm)	208.39	221.76	235.41	249.31	263.46	277.84

A. Calculate the average rate of change in the distance that thepebble fell during each of the following time intervals.

1) $6.0 \le t \le 6.4$ **2)** $6.2 \le t \le 6.4$ **3)** $6.4 \le t \le 7.0$

following interval

$$ARC = \frac{\Delta d}{\Delta t}$$
$$= \frac{235.41 - 208.39}{6.4 - 6.0}$$
$$= \frac{27.02}{0.4}$$
$$= 67.55 \text{ cm/s}$$

$\doteq 68.25$ cm/s $\doteq 70.716$ cm/s

after definitions on next screen

preceding interval

4) $6.4 \le t \le 6.8$ **5)** $6.4 \le t \le 6.6$

$\doteq 70.125$ cm/s $\doteq 69.5$ cm/s

B The ***instantaneous rate of change*** in the distance at t = 6.4 s is about 69 cm/s.
Look at smallest intervals using 6.4 as an endpoint.

C $6.2 \le t \le 6.6$
$= 68.875$ m/s

centred interval

after definitions on next screen

D It is the place where we want to know the instantaneous rate of change.

E By seeing the rate of change on both sides of 6.4, it will be easier to guess the rate of change at 6.4 because it will need to be somewhere in between those calculations. The best estimates come from the smallest intervals on either side of 6.4 s.

F 6.4 s is the midpoint of this interval. It balances the estimation error that results when only a single interval is used on either side of the value (above or below) for which the instantaneous rate of change is to be determined.

G No. I am able to guess what it might be but there are no values to the right of t = 7.0 to verify that the rate of change chosen might be correct.

2.2 Estimating Instantaneous Rates of Change from T of V and Equations Part1-f17

We now understand the concept of the average rate of change.

The ***instantaneous rate of change*** is the rate of change at one specific value ***x = a*** for a function ***y =f(x)***.
We begin by learning how to *approximate* it.

preceding interval
an interval of the independent variable of the form $a - h \le x \le a$, where h is a small positive value; used to determine an average rate of change

following interval
an interval of the independent variable of the form $a \le x \le a + h$, where h is a small positive value; used to determine an average rate of change

HW pp. 85-88 #1, 2a, 3, 8, 14

Ex.1:
A bacteria culture is growing exponentially and the population, *P*, is given by $P(n) = 200(1.2)^n$, where *n* is the number of hours.
Estimate, to the nearest tenth, the instantaneous rate of change at 5 hours, by using:
a) at least three preceding intervals
b) at least three following intervals

$$\frac{\Delta}{\Delta}$$

$$\frac{\Delta P}{\Delta n} = \frac{P(\quad) - P(\quad)}{\quad - \quad}$$

n	P(n)
4.5	
4.9	
4.99	
5	
5.01	
5.1	
5.5	

preceding interval

$4.5 \le n \le 5$

$$\frac{\Delta P}{\Delta n} = \frac{P(5) - P(4.5)}{5 - 4.5}$$

$$\doteq \frac{497.7 - 454.3}{0.5}$$

$\doteq 86.72$ bacteria/h

$4.9 \le n \le 5$

$$\frac{\Delta P}{\Delta n} = \frac{P(5) - P(4.9)}{5 - 4.9}$$

$\doteq 89.91$ bacteria/h

$4.99 \le n \le 5$

$$\frac{\Delta P}{\Delta n} = \frac{P(5) - P(4.99)}{5 - 4.99}$$

$\doteq 90.65$ bacteria/h

following interval

$5 \le n \le 5.5$

$$\frac{\Delta P}{\Delta n} = \frac{P(5.5) - P(5)}{5.5 - 5}$$

$\doteq 94.99$ bacteria/h

$5 \le n \le 5.1$

$$\frac{\Delta P}{\Delta n} = \frac{P(5.1) - P(5)}{5.1 - 5}$$

$\doteq 91.56$ bacteria/h

$5 \le n \le 5.01$

$$\frac{\Delta P}{\Delta n} = \frac{P(5.01) - P(5)}{5.01 - 5}$$

$\doteq 90.81$ bacteria/h

$$\text{Estimate} \doteq \frac{90.65 + 90.81}{2}$$

$\doteq 90.7$

The *instantaneous rate of change* (IROC) **at 5 hours** is about 90.7 bacteria/h.