HW pp. 85-88 #1, 2a, 3, 8, 14

p. 86

3. A population of raccoons moves into a wooded area. At t months, the number of raccoons, $P(t)$, can be modelled using the equation $P(t) = 100 + 30t + 4t^2$.

a) Determine the population of raccoons at 2.5 months.

b) Determine the average rate of change in the raccoon population over the interval from 0 months to 2.5 months.

c) Estimate the rate of change in the raccoon population at exactly 2.5 months.

d) Explain why your answers for parts a), b), and c) are different.

a) $P(2.5) = 100 + 30(2.5) + 4(2.5)^2$

$= 100 + 75 + 25$

$= 200$

b) $0 \le t \le 2.5$

$P(0) = 100 + 0 + 0$

$= 100$

aroc $= m_{secant}$

$= \dfrac{P(2.5) - P(0)}{2.5 - 0}$

$= \dfrac{200 - 100}{2.5}$

$= \dfrac{100}{2.5}$

$= 40$

c) $2.499 \le t \le 2.5$

$P(2.499) = 199.950004$

aroc $= m_{secant}$

$= \dfrac{P(2.5) - P(2.499)}{2.5 - 2.499}$

$= \dfrac{200 - 199.950004}{0.001}$

$= 49.996$

d) The answer for a) is different because it is NOT a rate of change.
It is the actual population at 2.5 months.

b) and c) differ, and c is much more accurate,
because the interval for the calculation in part b) is much larger
than the interval used for part c)

p. 87 **8.** Jacelyn purchased a new car for \$18 999. The yearly depreciation of the value of the car can be modelled by the equation $V(t) = 18\ 999(0.93)^t$, where $V(t)$ is the value of the car and t is the number of years that Jacelyn owns the car. Estimate the instantaneous rate of change in the value of the car when the car is 5 years old. What does this mean?

interval: $4.999 \le t \le 5.001$

$$m_{secant} = \frac{V(5.001) - V(4.999)}{5.001 - 4.999}$$

$$= \frac{18999(0.93)^{5.001} - 18999(0.93)^{4.999}}{0.002}$$

$$\doteq -959.194$$

$$\doteq -959.19$$

$\therefore$ at 5 years, the value of the car is depreciating at a rate of \$959.19 per year.

2.2 Estimating Instantaneous Rates of Change from Tables of Values and Equations (Part 2)

Math Learning Target:
"I can determine a reasonable estimate for the instantaneous rate of change using the methods presented, and interpret the result."

centred interval
an interval of the independent variable of the form $a - h \le x \le a + h$, where h is a small positive value; used to determine an average rate of change

(3 methods)

HW p.86 #2bc, 4a* only use centred intervals

Ex. 1: *(from yesterday)*
A bacteria culture is growing exponentially and the population, *P*, is given by $P(n) = 200(1.2)^n$, where n is the number of hours. *Estimate,* to the nearest tenth, the instantaneous rate of change at 5 hours, by using a centred interval.

Recall:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

centred interval

$4.999 \le n \le 5.001$

aroc = m_{secant}

$$\frac{\Delta P}{\Delta n} = \frac{P(5.001) - P(4.999)}{5.001 - 4.999}$$

$$= \frac{200(1.2)^{5.001} - 200(1.2)^{4.999}}{0.002}$$

$$\doteq \frac{0.181}{0.002}$$

$$\doteq 90.73$$

$$\doteq 90.7$$

it appears the populaton is growing at exactly 5 hours by about 90.7 bacteria per hour.

Calculating the **EXACT** Instantaneous Rate of Change

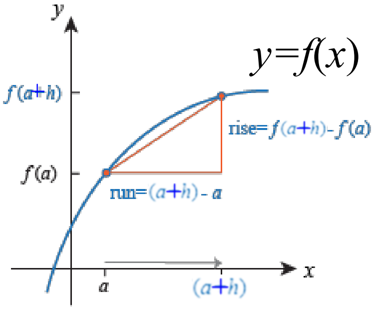
Math Learning Target:
"I can calculate the **exact** instantaneous rate of change by using the 'first principles' method."

(if time)
GeoGebra 2.2_2 Secant to Tangent Demo

$$\text{aroc} = m_{secant}$$
$$= \frac{f(a+h)-f(a)}{a+h-a}$$
$$= \frac{f(a+h)-f(a)}{h}$$

Find the exact value at $x=a$

$$\text{iroc} = m_{tangent}$$
$$= \frac{f(a+h)-f(a)}{h}, \text{ as } h \to 0$$



Ex.1: (Homework p.86 #3c)
Use 'first principles' to calculate the **exact** instantaneous rate of change at exactly 2.5 months. $t = 2.5$

$$P(t) = 100 + 30t + 4t^2$$
$$= 4t^2 + 30t + 100$$

$$\text{iroc} = m_{tangent}$$
$$= \frac{P(t+h)-P(t)}{h}, \text{ as } h \to 0$$
$$= \frac{[4(t+h)^2+30(t+h)+100]-[4t^2+30t+100]}{h}, \text{ as } h \to 0$$
$$= \frac{[4(2.5+h)^2+30(2.5+h)+100]-[4(2.5)^2+30(2.5)+100]}{h}, \text{ as } h \to 0$$
$$= \frac{[4(6.25+5h+h^2)+75+30h+100]-[4(6.25)+75+100]}{h}, \text{ as } h \to 0$$
$$= \frac{[4h^2+20h+25+30h+175-[25+175]}{h}, \text{ as } h \to 0$$
$$= \frac{4h^2+50h}{h}, \text{ as } h \to 0$$

* constant term disappears

$$= \frac{\cancel{h}(4h+50)}{\cancel{h}}, \text{ as } h \to 0$$
$$= 4h+50, \text{ as } h \to 0$$
$$= 4(0)+50$$
$$= 50$$

at 2.5 months, the racoon populaton is increasing by exactly 50 racoons per month.

For each question, do NOT estimate the rate of change.
Rather, find the exact rate of change using 'first principles'.

pp. 86-87 #4c, 5, 10* do not approximate π

Challenge: For the function $y = \frac{1}{x}$ find the exact rate of change at $x = 2$.