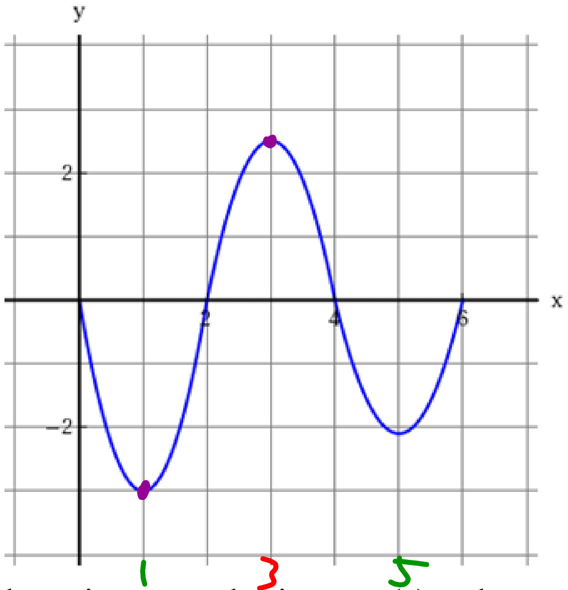
MHF 4UI: Local and Global Extrema Practice

Given a function $y = f(x)$ defined on $0 \le x \le 6$:



For this function's domain, state the integer(s) x that correspond to a…

Local minimum	Local maximum	Global minimum	Global maximum
x=1, 5	x=3	x=1	x=3

Mon. Sept. 25 pp.103-106 #1, 2*, 3 to 9*, 10, 11, 14
* in #2, the answer in the back has a small error.
Do you know what it is? Also, the answer for #9 in the back has some mistakes.

Tues. Sept. 26 pp.116-117 #2, 3, 5*an estimate is required only,
6a*find the quadratic equation first then use the preceding interval method,
8, 9, 10, 11ab*use the algebraically simplified DQ, 13

p.118 (45 minutes max) #1,2,3,4a* use the algebraically simplified DQ

Wed. Sept. 27 **Use the ALGEBRAICALLY SIMPLIFIED DIFFERENCE QUOTIENT FOR ALL RATE OF CHANGE CALCS**

pp.111-113 #1, 3, 4, 6c, 9a, ~~10~~, 14

5c

pp.116 #5

5. The height, in centimetres, of a piston attached to a turning wheel at time t, in seconds, is modelled by the equation $y = 2 \sin(120° t)$.

a) Examine the equation, and select a strategy for finding the instantaneous rate of change in the piston's height at $t = 12$ s.

5a $y = 2\sin(120°t)$ $t = 12$ $\therefore$ use 11.999

$$aroc = m_{secant}$$

$$= \frac{f(12) - f(11.999)}{12 - 11.999}$$

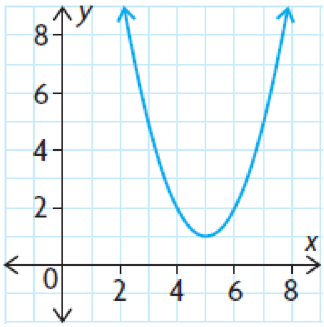
$$= \frac{2\sin(120(12)) - 2\sin(120(11.999))}{0.001}$$

$$= \frac{0.004}{0.001}$$

$$\doteq 4.188$$

pp.116 #6

6. For the graph shown, estimate the slope of the tangent line at each point.



a) (4, 2)
b) (5, 1)
c) (7, 5)

(a) Eq'n : $y=(x-5)^2+1$ @ (4,2) preceding (3,5)

preceding $3.999 \le x < 4$

$$\frac{f(4)-f(3.999)}{4-3.999}$$

$$=\frac{((4-5)^2+1)-((3.999-5)^2+1)}{0.001}$$

$$=\frac{2-2.002001}{0.001}$$

$$=\frac{-0.002001}{0.001}$$

$$=-2.001$$

p.111

1. $C(x) = 0.3x^2 - 0.9x + 1.675$ @ $x = 1.5$ (1500)

$iroc = m_{tangent}$

$= \dfrac{C(1.5+h) - C(1.5)}{h}$, as $h \to 0$

$= \dfrac{[0.3(1.5+h)^2 - 0.9(1.5+h) + 1.675] - [0.3(1.5)^2 - 0.9(1.5) + 1.675]}{h}$, as $h \to 0$

$= \dfrac{[0.3(2.25 + 3h + h^2) - 1.35 - 0.9h + 1.675] - [0.675 - 1.35 + 1.675]}{h}$, as $h \to 0$

$= \dfrac{0.675 + 0.9h + 0.3h^2 - 0.9h + 0.325 - (1)}{h}$ as $h \to 0$

$= \dfrac{0.3h^2}{h}$, as $h \to 0$

$= 0.3h$, as $h \to 0$

$\therefore$ As $h \to 0$, $iroc \to 0$

$\therefore$ at $x = 1.5$ (1500), $m_{tangent} = 0$

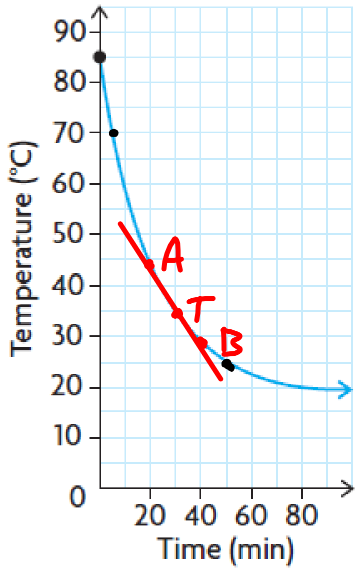
$\therefore$ local max or min.

$h<0$	$h=0$	$h>0$
$-$	0	$+$
\	—	/

minimum $C(x)$

$\therefore$ 1500 items is local min

↳ most economical production level



2. A cup of hot cocoa left on a desk in a classroom had its temperature measured once every minute. The graph shows the relationship between the temperature of the cocoa, in degrees Celsius, and time, in minutes.

a) Determine the slope of the secant line that passes through the points (5, 70) and (50, 25).

b) What does the answer to part a) mean in this context?

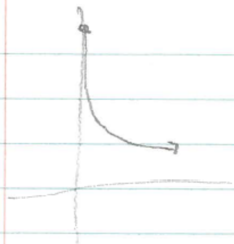
c) Estimate the slope of the tangent line at the point (30, 35).

d) What does the answer to part b) mean in this context?

e) Discuss what happens to the rate at which the cup of cocoa cools over the 90 min period.

p.118

2.



a) Secant (5,70), (50,25)

$aroc = \frac{25-70}{50-5}$

$= \frac{-45}{45}$

$= -1$

b) the aroc between 5 min & 50 min is 1° decrease in temp. per min

c) Estimate m_t at $T(30,35)$

try $A(20,45)$ and $B(40,30)$?

$\therefore m_{AT} = \frac{35-45}{30-20}$ $\qquad m_{TB} = \frac{30-35}{40-30}$

$= \frac{-10}{10}$ $\qquad = \frac{-5}{10}$

$= -1$ $\qquad = -0.5$

$Avg = \frac{-1 + (-0.5)}{2}$

$= -0.75$

d) at (30,35) it's cooling at 0.75°C / min after 30 min.

e) ∴ the rate of decrease, decreases over the entire interval, until it is nearly 0, and constant (at room temperature).

$$(x+y)^2$$
$$= x^2 + 2xy + y^2$$

$$(x-y)^2$$
$$= x^2 - 2xy + y^2$$

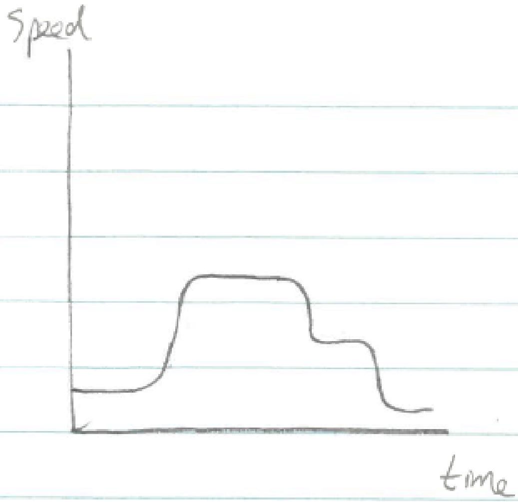
$$(4xz^2 - 3y^4)^2$$
$$= 16x^2z^4 - 24xz^2y^4 + 9y^8$$

$$(x-y)^3$$
$$= 1x^3 - 3x^2y + 3xy^2 - 1y^3$$

$$(x+y)^8$$

MHF 4UI 2.4

p.105 9.



p.111 #6

6c) $f(x) = x^2 - 9x \quad (4.5, -20.25)$

$m_{tangent} = iroc$

$= \dfrac{f(4.5+h) - f(4.5)}{h}$, as $h \to 0$

$= \dfrac{(4.5+h)^2 - 9(4.5+h) - [(4.5)^2 - 9(4.5)]}{h}$, as $h \to 0$

$= \dfrac{20.25 + 9h + h^2 - 40.5 - 9h - [20.25 - 40.5]}{h}$, as $h \to 0$

$= \dfrac{h^2 - 20.25 - (-20.25)}{h}$, as $h \to 0$

$= \dfrac{h^2}{h}$, as $h \to 0$

$= h$, as $h \to 0$

As $h \to 0$, $m_t = 0$ ∴ local max or min

$h<0$	$h=0$	$h>0$
$-$	0	$+$
\	—	/

∴ local minimum

p.111 #10

10. $h(t) = -5t^2 + 5t + 10$; max at $t = 0.5$

$\text{iroc} = m_{\text{tangent}}$

$= \dfrac{f(0.5+h) - f(0.5)}{h}$, as $h \to 0$

$= \dfrac{[-5(0.5+h)^2 + 5(0.5+h) + 10] - [-5(0.5)^2 + 5(0.5) + 10]}{h}$, as $h \to 0$

$= \dfrac{[-5(0.25+h+h^2) + 2.5 + 5h + 10] - [-1.25 + 2.5 + 10]}{h}$, as $h \to 0$

$= \dfrac{-1.25 - 5h - 5h^2 + 5h + 12.5 - (11.25)}{h}$, as $h \to 0$

$= \dfrac{-5h^2}{h}$, as $h \to 0$

$= -5h$

$\therefore$ as $h \to 0$, $\text{iroc} \to 0$

$\therefore$ at $t = 0.5$, $m_t = 0$

$\therefore$ local max or min.

$h<0$	$h=0$	$h>0$
+	0	−
/	—	\

$\therefore$ local maximum.

p.118 #4

4a) $h(p) = 2p^2 + 3p \quad p = -1$

$iroc = \frac{f(-1+h) - f(-1)}{h}$

$= \frac{2(-1+h)^2 + 3(-1+h) - [2(-1)^2 + 3(-1)]}{h}$, as $h \to 0$

$= \frac{2(1 - 2h + h^2) - 3 + 3h - [2-3]}{h}$, as $h \to 0$

$= \frac{2 - 4h + 2h^2 - 3 + 3h + 1}{h}$, as $h \to 0$

$= \frac{2h^2 - h}{h}$ as $h \to 0$

$= 2h - 1$ as $h \to 0$

$= -1$

$p = -0.75$

$= \frac{2(-0.75+h)^2 + 3(-0.75+h) - [2(-0.75)^2 + 3(-0.75)]}{h}$, as $h \to 0$

$= \frac{2(0.5625 - 1.5h + h^2) - 2.25 + 3h - [1.125 - 2.25]}{h}$, as $h \to 0$

$= \frac{1.125 - 3h + 2h^2 - 2.25 + 3h - (-1.125)}{h}$, as $h \to 0$

$= \frac{2h^2}{h}$, as $h \to 0$

$= 2h$, as $h \to 0$

$= 0$

$h<0$	$h=0$	$h>0$
$-$	0	$+$
\	—	/

$\therefore p = -0.75$ is local min.

$p = 1$

$iroc = f(1+h) - f(1)$

$= \frac{2(1+h)^2 + 3(1+h) - [2(1)^2 + 3(1)]}{h}$ as $h \to 0$

$= \frac{2(1 + 2h + h^2) + 3 + 3h - [5]}{h}$, as $h \to 0$

$= \frac{2 + 4h + 2h^2 + 3h - 2}{h}$, as $h \to 0$

$= \frac{2h^2 + 7h}{h}$, as $h \to 0$

$= 2h + 7$, as $h \to 0$

$= 7$

Extra p.111 5c

$f(x) = 5\sin x$ at 90°

$irOC = m_{tangent}$

$= \frac{f(90+h) - f(90)}{h}$, as $h \to 0$

$= \frac{5\sin(90+h) - 5\sin 90}{h}$, as $h \to 0$

$= \frac{5[\sin 90 \cos h + \cos 90 \sin h] - 5\sin 90^\circ}{h}$, as $h \to 0$

$\sin(x+y)$

$= \sin x \cos y + \cos x \sin y$