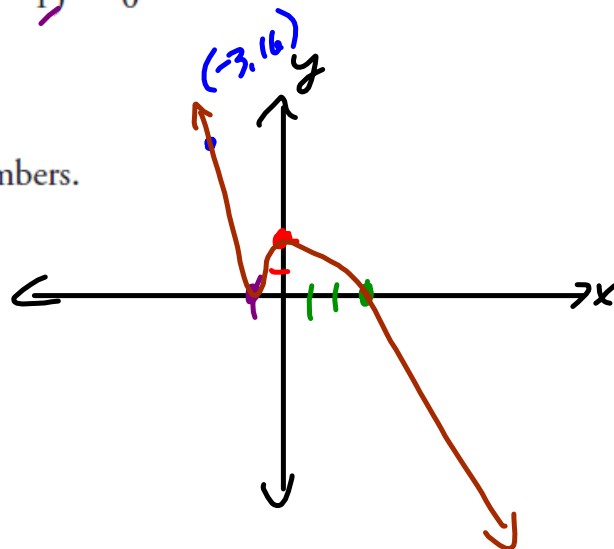


pp. 127-128 # 1, 2, 3d, 5, 7, 8

5. Draw a graph of a polynomial function that satisfies all of the following characteristics:

- $f(-3) = 16$, $f(3) = 0$, and $f(-1) = 0$
- The y -intercept is 2. ✓
- $f(x) \geq 0$ when $x < 3$.
- $f(x) \leq 0$ when $x > 3$.
- The domain is the set of real numbers.



3.2 Characteristics of Polynomial Functions



Math Learning Target:

"I can identify properties of any polynomial function."

A **leading coefficient** is the coefficient of the term with the highest exponent for powers of x in the polynomial expression or function.

For example, 4 is the leading coefficient in the polynomial function:

$$f(x) = -2x + 7 + 4x^3$$

A **turning point** is a point on a curve where the relation changes from increasing to decreasing, and vice versa. (For an example see p. 30)

An **absolute maximum** is synonymous with global maximum.

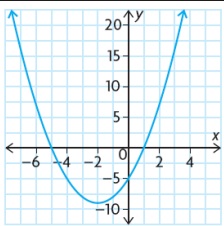
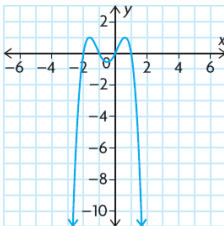
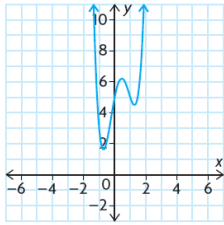
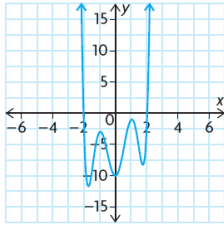
An **absolute minimum** is synonymous with global minimum.

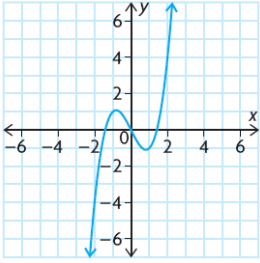
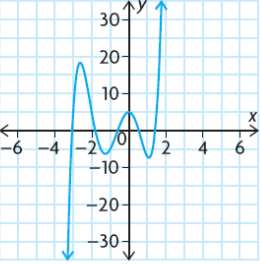
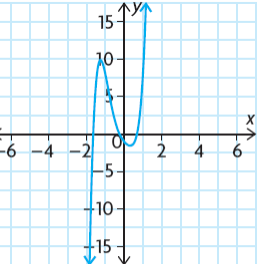
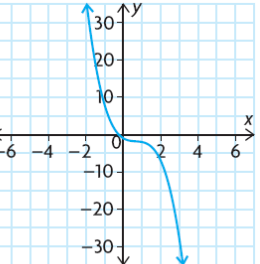
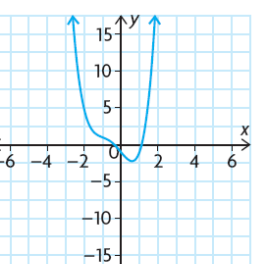
(For an example see p. 131)

INVESTIGATE the Math. pp. 129-131 **A-E** and **G-M**. Use [desmos](#)

A chart for parts A and E has already been created for you.

Answer the rest of the questions in your notebook.

A.	Degree	Even or Odd Degree	Leading Coefficient	End Behaviours		Number of turning points	
				$x \rightarrow -\infty$	$x \rightarrow +\infty$		
a)		2	even	1	$y \rightarrow \infty$	$y \rightarrow \infty$	1
b)							
c)							
d)							

	Degree	Even or Odd Degree	Leading Coefficient	End Behaviours		Number of turning points
				$x \rightarrow -\infty$	$x \rightarrow +\infty$	
e)						
	$f(x) = x^3 - 2x$					
f)						
	$f(x) = 2x^5 + 7x^4 - 3x^3 - 18x^2 + 5$					
g)						
	$f(x) = 5x^5 + 5x^4 - 2x^3 + 4x^2 - 3x$					
h)						
	$f(x) = -2x^3 + 4x^2 - 3x - 1$					
i)						
	$f(x) = x^4 + 2x^3 - 3x - 1$					

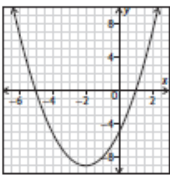
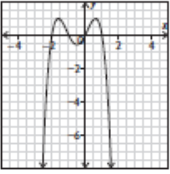
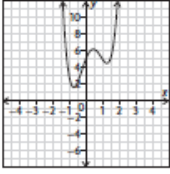
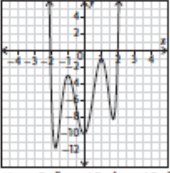
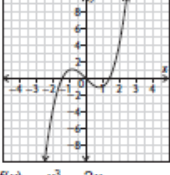
E.	<u>Even Functions</u>	<u>Odd Functions</u>	Neither
i) $f(x) = x^4 - 2x^2 + 1$			

Read and **STUDY** p.135

Complete pp. 136-138 #1ab, 2ab, 3, 4abf, 5, 7ad, 10, 13, 14, 16

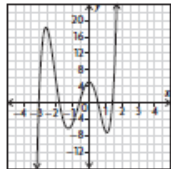
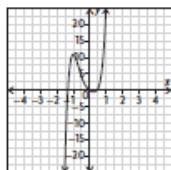
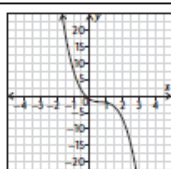
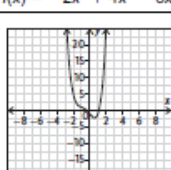
Answers to Investigate the Math

A.

Equation and Graph	Degree	Even or Odd Degree?	Leading Coefficient	End Behaviours		Number of Turning Points
				$x \rightarrow -\infty$	$x \rightarrow +\infty$	
a)  $f(x) = x^2 + 4x - 5$	2	even	+1	$y \rightarrow +\infty$	$y \rightarrow +\infty$	1
b)  $f(x) = -x^4 - 2x^3 + x^2 + 2x$	4	even	-1	$y \rightarrow -\infty$	$y \rightarrow -\infty$	3
c)  $f(x) = 3x^4 - 4x^3 - 4x^2 + 5x + 5$	4	even	3	$y \rightarrow +\infty$	$y \rightarrow +\infty$	3
d)  $f(x) = 2x^6 - 12x^4 + 18x^2 + x - 10$	6	even	2	$y \rightarrow +\infty$	$y \rightarrow +\infty$	5
e)  $f(x) = x^3 - 2x$	3	odd	1	$y \rightarrow -\infty$	$y \rightarrow +\infty$	2

Answers to Investigate the Math (Continued)

A.

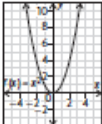
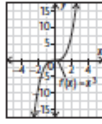
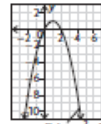
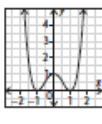
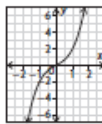
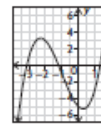
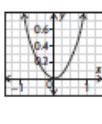
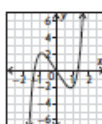
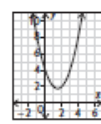
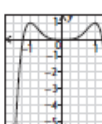
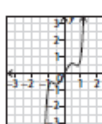
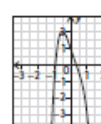
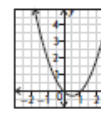
Equation and Graph	Degree	Even or Odd Degree?	Leading Coefficient	End Behaviours		Number of Turning Points
				$x \rightarrow -\infty$	$x \rightarrow +\infty$	
f)  $f(x) = 2x^5 + 7x^4 - 3x^3 - 18x^2 + 5$	5	odd	2	$y \rightarrow -\infty$	$y \rightarrow +\infty$	4
g)  $f(x) = 5x^5 + 5x^4 - 2x^3 + 4x^2 - 3x$	5	odd	5	$y \rightarrow -\infty$	$y \rightarrow +\infty$	2
h)  $f(x) = -2x^3 + 4x^2 - 3x - 1$	3	odd	-2	$y \rightarrow +\infty$	$y \rightarrow -\infty$	0
i)  $f(x) = x^4 + 2x^3 - 3x - 1$	4	even	1	$y \rightarrow +\infty$	$y \rightarrow +\infty$	1

B. Answers may vary. Odd degrees have opposite end behaviour; even degrees have identical end behaviour. If the leading coefficient is positive and the degree is even, $y \rightarrow +\infty$ in both directions, whereas if the leading coefficient is negative, $y \rightarrow -\infty$ in both directions. In an odd degree polynomial, if the leading coefficient is positive, then as $x \rightarrow +\infty$, $y \rightarrow +\infty$ and as $x \rightarrow -\infty$, $y \rightarrow -\infty$, and vice versa for a negative leading coefficient. For a polynomial of degree n , there are at most $n - 1$ turning points.

C. Answers may vary; but the new polynomials should still meet the above observations.

D. $n - 1$

E.

Even Functions	Odd Functions	Neither Even nor Odd Functions
(symmetry in the y-axis) $f(x) = x^2$ 	(rotational symmetry around the origin) $f(x) = x^3$ 	(neither of these symmetries) $f(x) = -x^2 + 2x$ 
$f(x) = x^4 - 2x^2 + 1$ 	$f(x) = x^3 + x$ 	$f(x) = x^3 + 3x^2 - 2x - 5$ 
$f(x) = \frac{1}{2}x^{10} - \frac{1}{3}x^4 + x^2$ 	$f(x) = x^5 - 3x$ 	$f(x) = x^2 - 3x + 4$ 
$f(x) = -2x^6 + 3x^4$ 	$f(x) = 2x^7 - 3x^3 + 2x$ 	$f(x) = -3x^4 + 2x^3 - 3x + 1$ 
		$f(x) = x^2 - x$ 

G. No, for example, ix).

H. No, for example, ii).

I. Polynomials of odd degree have opposite end behaviour; polynomials of even degree have identical end behaviour. If the leading coefficient is positive and the degree is even, $y \rightarrow +\infty$ in both directions, whereas if the leading coefficient is negative, $y \rightarrow -\infty$ in both directions. In an odd degree polynomial, if the leading coefficient is positive, then as $x \rightarrow +\infty$, $y \rightarrow +\infty$ and as $x \rightarrow -\infty$, $y \rightarrow -\infty$, and vice versa for a negative leading coefficient. For a polynomial of degree n , there are at most $n - 1$ turning points. To check symmetry, check $f(-x)$.

Answers to Reflecting

J. They have identical end behaviour on either side.

K. Because their end behaviour is opposite, which means the graph must pass through the x -axis at least once.L. Yes, for example, $f(x) = x^2 + 1$.M. It is not possible to predict it precisely, but if the degree is n , the number of zeros is at most n , and a polynomial of odd degree must have at least one real zero.