

Unit 4: Polynomial Equations and Inequalities

4.1 Solving Polynomial Equations PART 1

**Math Learning Target:**

"By the end of next class, I can solve any polynomial equation."

Ex. 1: Find the point(s) of intersection of these polynomial functions algebraically:

$$y = 2x^4 + x^3 + x^2 - 7x - 20 \quad \textcircled{1}$$

$$y = x^4 - 3x^3 - x^2 + 6x + 10 \quad \textcircled{2}$$

Sub \textcircled{1} in \textcircled{2}

$$2x^4 + x^3 + x^2 - 7x - 20 = x^4 - 3x^3 - x^2 + 6x + 10$$

$$2x^4 - x^4 + x^3 + 3x^3 + x^2 + x^2 - 7x - 6x - 20 - 10 = 0$$

$$x^4 + 4x^3 + 2x^2 - 13x - 30 = 0$$

$$P(x) = x^4 + 4x^3 + 2x^2 - 13x - 30$$

$$P(2) = 0 \therefore x-2 \text{ is a factor}$$

$$\begin{array}{r} 2 | 1 & 4 & 2 & -13 & -30 \\ \downarrow & 2 & 12 & 28 & 30 \\ 1 & 6 & 14 & 15 & 0 \end{array}$$

$$P(x) = (x-2)(x^3 + 6x^2 + 14x + 15)$$

$$P(x) = (x-2)g(x)$$

$$g(-3) = 0 \therefore x+3 \text{ is a factor}$$

if $x=2$, Sub in \textcircled{2}

$$y=(2)^4 - 3(2)^3 - (2)^2 + 6(2) + 10$$

= 10 $\therefore (2, 10)$ is a P.O.I.

if $x=-3$

$$y=(-3)^4 - 3(-3)^3 - (-3)^2 + 6(-3) + 10$$

$$= 145$$

$\therefore (-3, 145)$ is a P.O.I.

$$y = 2x^4 + x^3 + x^2 - 7x - 20$$

$$y = x^4 - 3x^3 - x^2 + 6x + 10$$

$$\begin{array}{r} -3 | 1 & 6 & 14 & 15 \\ \downarrow & -3 & -9 & -15 \\ 1 & 3 & 5 & 0 \end{array}$$

$$\therefore P(x) = (x-2)(x+3)(x^2+3x+5)$$

Check if x^2+3x+5 factors.

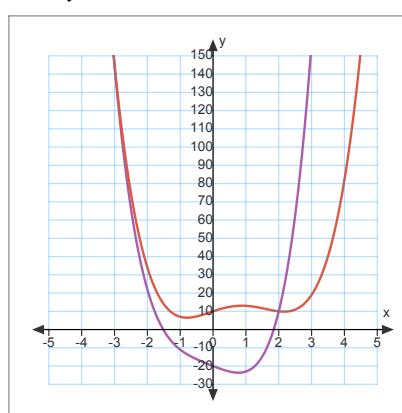
discriminant b^2-4ac

$$= (3)^2 - 4(1)(5)$$

$$= -11$$

$\therefore b^2-4ac < 0$

$\therefore x^2+3x+5$ has no real roots.



Does $2 = 1$?

Step 1. Let a and b be equal. Note: assume a and b are not zero.

Step 2. Multiply the equation by a .

Step 3. Subtract the equation by b^2 .

Step 4. Factor the equation

Step 5. Divide the equation by $(a - b)$.

Step 6. Remember Step 1?

Step 7. Divide the equation by a .

$$\begin{aligned}
 a &= b \\
 a^2 &= ab \\
 a^2 - b^2 &= ab - b^2 \\
 \frac{(a+b)(a-b)}{a-b} &= \frac{b(a-b)}{a-b} \\
 a+b &= b \\
 a+(a) &= (a) \\
 2a &= a \\
 \frac{2a}{a} &= \frac{a}{a} \\
 2 &= 1
 \end{aligned}$$

Below is an example on the left of where the above misconception has been demonstrated by students:

Solve: $x^2 - 4x = 0$

$$\begin{aligned}
 x^2 &= 4x \\
 \frac{x^2}{x} &= \frac{4x}{x} \\
 x &= 4
 \end{aligned}$$

Solve: $x^2 - 4x = 0$

$$\begin{aligned}
 x(x-4) &= 0 \\
 \downarrow & \quad \downarrow \\
 x=0 & \quad x-4=0 \\
 & \quad x=4
 \end{aligned}$$

Ex. 2: Solve $\{x \in \mathbb{R}\}$.

a) $2x^3 - 240x = 0$

$$\begin{aligned} 2x(x^2 - 120) &= 0 \\ \downarrow & \quad \downarrow \\ 2x = 0 & \quad x^2 - 120 = 0 \\ x = 0 & \quad x^2 = 120 \\ & \quad x = \pm\sqrt{120} \\ & \quad = \pm\sqrt{4\sqrt{30}} \\ & \quad = \pm 2\sqrt{30} \end{aligned}$$

b) $(x+2)(10x^2 - 19x - 15) = 0$

$$\begin{aligned} (x+2)(2x-5)(5x+3) &= 0 \\ \downarrow & \quad \downarrow & \quad \downarrow \\ x+2=0 & \quad 2x-5=0 & \quad 5x+3=0 \\ x=-2 & \quad x=\frac{5}{2} & \quad x=-\frac{3}{5} \end{aligned}$$

$$\begin{aligned} b^2 - 4ac & \\ = (-19)^2 - 4(10)(-15) & \\ = 361 + 600 & \\ = 961 & \\ \because b^2 - 4ac > 0 & \left\{ \begin{array}{l} \sqrt{961} \\ = 31 \end{array} \right. \\ \therefore \text{it has real roots} & \left. \begin{array}{l} \therefore \text{it} \\ \text{factors} \end{array} \right. \end{aligned}$$

Today's Entertainment: p. 204 #1, 2, 3, 5, 6.

*For #2 you do not have to verify using technology.

Also for #2d one of the roots is -3 (not 3)