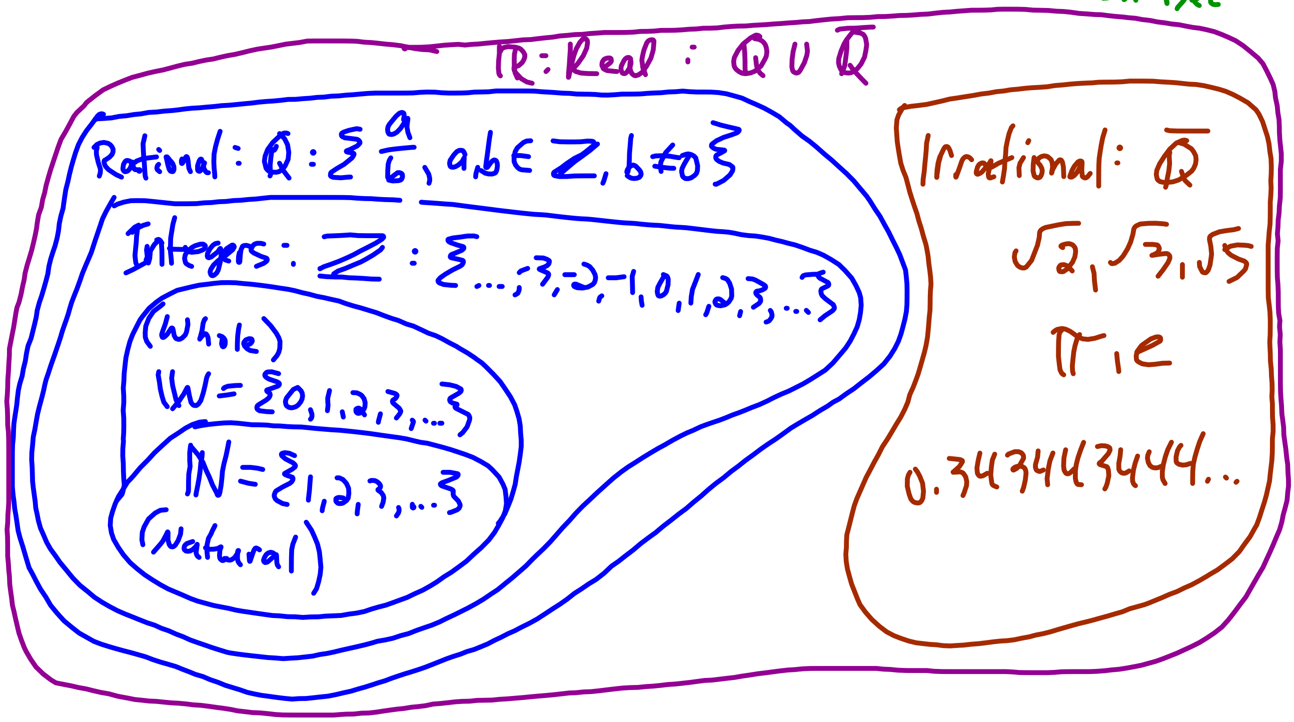
Number Systems

$\cup$ (or) union { $\cap$ ← Intersec (and)



Rational Zeros Theorem

If P(x) is a polynomial with integer coefficients,

and if $\frac{p}{q}$ is a zero of P(x), i.e. $P\left(\frac{p}{q}\right) = 0$

then p is a factor of the constant term of P(x)
and q is a factor of the leading coefficient of P(x).

Lesson 3.6_2 Ex.1

$$x^4 - 2x^3 - 7x^2 + 8x + 12$$

$p = 12$ $\pm\left(\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{6}{1}, \frac{12}{1}\right)$

$q = 1$

Ans: $(x-2)(x+2)(x-3)(x+1)$

vs

Lesson 4.1_1 Ex.2b

$$(x+2)(10x^2 - 19x - 15) = 0$$

$p = 15: 1, 3, 5, 15$

$q = 10: 1, 2, 5, 10$

$\pm\left(\frac{1}{1}, \frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{3}{1}, \frac{3}{2}, \frac{3}{5}, \frac{3}{10}, \frac{5}{1}, \frac{5}{2}, \cancel{\frac{5}{5}}, \cancel{\frac{5}{10}}, \frac{15}{1}, \frac{15}{2}, \cancel{\frac{15}{5}}, \cancel{\frac{15}{10}}\right)$

$1 \quad \frac{1}{2} \quad 3 \quad \frac{3}{2}$

Ans: $(x+2)(2x+5)(5x-3) = 0$

New

$$6x^3 + 41x^2 - 8x - 7 = 0$$

$1, 7$

$1, 2, 3, 6$

$\pm\left(\frac{7}{1}, \frac{1}{1}, \frac{7}{2}, \frac{1}{2}, \frac{7}{3}, \frac{1}{3}, \frac{7}{6}, \frac{1}{6}\right)$

4.2 Solving Linear Inequalities

Math Learning Target:

"By the end of class, I can solve any linear inequality."

Recall:

$=$ Equal

$<$ Less than

$>$ Greater

$\leq$ Less than or Equal

$\geq$

$\neq$ Not Equal

Ex. 1: Consider the inequality 12 > 9...

12 > 9	12 > 9	12 > 9	12 > 9	12 > 9	12 > 9
+4	+4	-3	÷3	÷(-3)	×(-2)
$16 > 13$	$9 > 6$	$4 > 3$	$\frac{12}{-3} > \frac{9}{-3}$	$-24 > -18$	
True	True	True	$-4 > -3$	False	
			False	$-24 < -18$	
			$\therefore -4 < -3$		

Rule:

Whenever you **multiply or divide an inequality** by a **negative** number, you **MUST reverse the inequality sign** to preserve the validity.

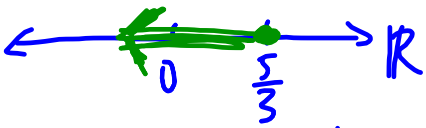
A **linear inequality** is an inequality that contains algebraic expression(s) that has (have) at most degree 1.

Ex. 2: Solve the linear inequality $\{x \in \mathbb{R}\}$.

$3x - 5 \leq 0$

$3x \leq 5$

$x \leq \frac{5}{3}$



$\{x \in \mathbb{R} \mid x \leq \frac{5}{3}\}$

$(-\infty, \frac{5}{3}]$

Ex. 3: Solve $\{x \in \mathbb{R}\}$.

$5 - 2x < -7 + x$

$-2x - x < -7 - 5$

$-3x < -12$

$x > \frac{-12}{-3}$

$x > 4$

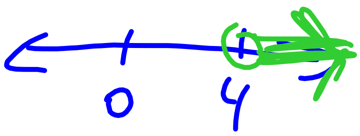
$5 - 2x < -7 + x$

$5 + 7 < x + 2x$

$12 < 3x$

$\frac{12}{3} < x$

$4 < x$



$\therefore \{x \in \mathbb{R} \mid x > 4\}$

$(4, \infty)$

Test values $x = 5$

$LS = 5 - 2x$ $RS = -7 + x$

$= 5 - 2(5)$ $= -7 + 5$

$= -5$ $= -2$

$LS < RS$ $\therefore$ true

$x = 3$

$LS = 5 - 2x$ $RS = -7 + x$

$= 5 - 2(3)$ $= -7 + 3$

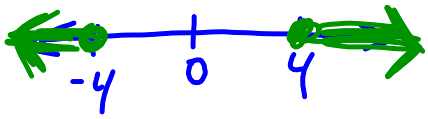
$= -1$ $= -4$

$LS > RS \rightarrow$ False

Doesn't Match Question

Ex. 4: Solve $\{x \in \mathbb{R}\}$. Express your final answer in interval notation.

$|x| \geq 4$



$(-\infty, -4] \cup [4, \infty)$

Ex. 5: Solve $29 \le 5(2x+3)-4(x+1) \le 71$

a) $x \in \mathbb{R}$

b) $x \in W$

c) $x \in \mathbb{Z}$

$29 \le 10x + 15 - 4x - 4 \le 71$

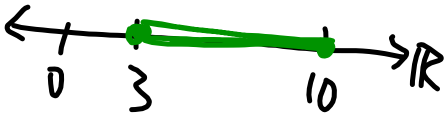
$29 \le 6x + 11 \le 71$

$29 - 11 \le 6x + 11 - 11 \le 71 - 11$

$18 \le 6x \le 60$

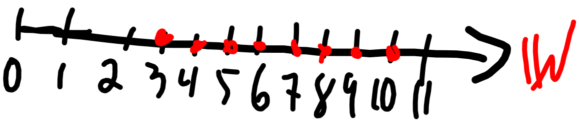
$3 \le x \le 10$

a)



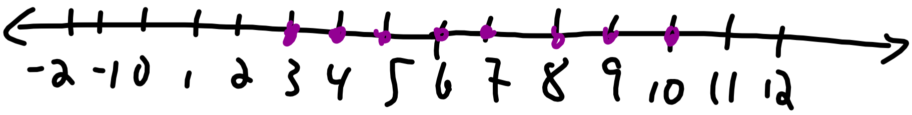
$\{x \in \mathbb{R} \mid 3 \le x \le 10\}$

b)



$\{3, 4, 5, 6, 7, 8, 9, 10\}$

c)



Entertainment: pp. 213-215 #2bc, 4f, 6d, 7ef, 9*, 12, 15 Challenge: #19
*answers may vary for 9b)

Entertainment: p. 204 # *8ac, 7b, 9c, 10, 11 , 13, 15, 16***, 18**
*** do #8 first ** means**
***x* is a Whole number**

*** wrong answer in back: it should be $x=5$, $x=-2$ and $x=-3$

18
16
13e, a
11c

p. 205

11. The Sickle-Lichti family members are very competitive card players. They keep score using a complicated system that incorporates positives and negatives. Maya's score for the last game night could be modelled by the function $S(x) = x(x-4)(x-6)$, $x < 10$, $x \in \mathbf{W}$, where x represents the game number.

a) After which game was Maya's score equal to zero?

b) After which game was Maya's score -5?

c) After which game was Maya's score 16?

$\therefore S(x) = 16$

$16 = x(x-4)(x-6)$

$0 = (x^2-4x)(x-6) - 16$

$P(x) = x^3 - 6x^2 - 4x^2 + 24x - 16$

$= x^3 - 10x^2 + 24x - 16$

$P(1) = -1$

$P(2) = 0$

$\therefore$ Maya's score was 16 after the second game.

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13. The distance of a ship from its harbour is modelled by the function $d(t) = -3t^3 + 3t^2 + 18t$, where t is the time elapsed in hours since departure from the harbour.

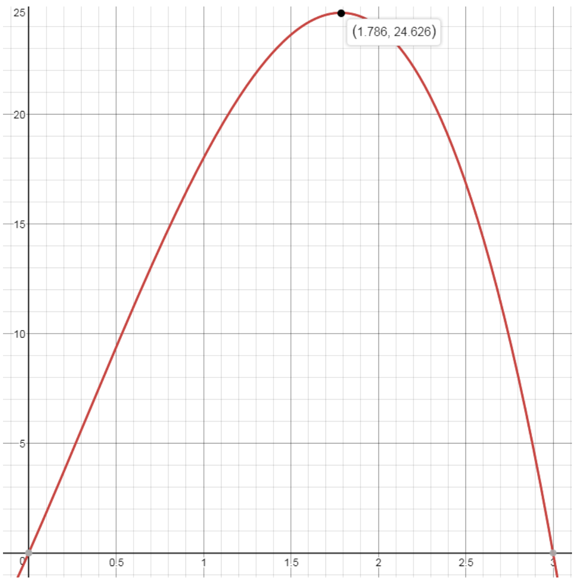
a) Factor the time function.
b) When does the ship return to the harbour?
c) There is another zero of $d(t)$. What is it, and why is it not relevant to the problem?
d) Draw a sketch of the function where $0 \leq t \leq 3$.
e) Estimate the time that the ship begins its return trip back to the harbour.

a) $d(t) = -3t^3 + 3t^2 + 18t$
$= -3t(t^2 - t - 6)$
$= -3t(t-3)(t+2)$

From a) if $d(t)=0$, then the ship is at the harbour.
$\therefore t = 0$ (hasn't left yet), $t = 3$ (ship back at the harbour)

Note $t=-2$ is inadmissible
$\because$ time must be ≥ 0.

e) based on the graph at right from part d)
the ship's max. distance from harbour is 24.6 km at time 1.786 hours
$\therefore$ it begins its return trip back at about 1.8 hours.



p. 206

16. Determine algebraically where the cubic polynomial function that has zeros at 2, 3, and −5 and passes through the point (4, 36) has a value of 120.

$y = a(x-2)(x-3)(x+5)$ thru $(4,36)$

$36 = a(4-2)(4-3)(4+5)$

$36 = a(2)(1)(9)$

$36 = 18a$

$\therefore a = 2$ $\therefore y = 2(x-2)(x-3)(x+5)$ is the equation

Now, when does it have a value of 120?

Let $y = 120$

$\therefore 120 = 2(x-2)(x-3)(x+5)$

$60 = (x-2)(x-3)(x+5)$

$= (x^2-5x+6)(x+5)$

$0 = x^3+5x^2-5x^2-25x+6x+30-60$

$P(x) = x^3-19x-30$

$P(-2) = (-2)^3-19(-2)-30$

$= -8+38-30$

$= 0$ $\therefore x+2$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -19 & -30 \\ & \downarrow & -2 & 4 & 30 \\ \hline & 1 & -2 & -15 & \cancel{0} \end{array}$$

$\therefore P(x) = (x+2)(x^2-2x-15)$

$= (x+2)(x-5)(x+3)$

$\therefore$ the polynomial has a value of 120 at $x=-2$, $x=5$ and $x=-3$.

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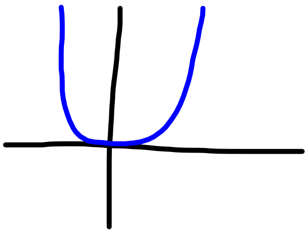
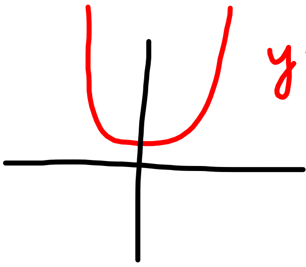
18. a) It is possible that a polynomial equation of degree 4 can have no real roots. Create such a polynomial equation and explain why it cannot have any real roots.

$y = x^4$





$y = x^4 + 1$

b) Explain why a degree 5 polynomial equation must have at least one real root.

b) A degree 5 polynomial function $y = f(x)$ has opposite end behaviour, so somewhere in the middle it must cross the x-axis. This means its corresponding equation $0 = f(x)$ will have at least one real root.

Ex.