

Entertainment: **Do #7 first...** pp.227-228 #7*ef, 3, 8, 9, 12**, 13**, 14, 15

Challenge: #18

* use **desmos** to confirm your answers;

** the text has answers rounded in the back, but you must state your answers as exact values

$$\text{cd} - b -$$

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12. A rock is tossed from a platform and follows a parabolic path through the air. The height of the rock in metres is given by

$$h(t) = -5t^2 + 12t + 14, \text{ where } t \text{ is measured in seconds.}$$

a) How high is the rock off the ground when it is thrown?

b) How long is the rock in the air?

c) For what times is the height of the rock greater than 17 m?

d) How long is the rock above a height of 17 m?

c) $h(t) > 17$

$$\therefore -5t^2 + 12t + 14 > 17$$

$$-5t^2 + 12t + 14 - 17 > 0$$

$$-5t^2 + 12t - 3 > 0$$

$$\therefore h(t) = 17$$

$$\text{if } t = \frac{6+\sqrt{21}}{5} \text{ or } t = \frac{6-\sqrt{21}}{5}$$

$\therefore h(t) > 17$ between these 2 times.

$$\therefore \left\{ t \in \mathbb{R} \mid \frac{6-\sqrt{21}}{5} < t < \frac{6+\sqrt{21}}{5} \right\}$$

$$\left\{ t \in \mathbb{R} \mid 0.28 < t < 2.11 \right\}$$

d) how long above 17 m
is the difference between the two times.

$$\therefore \frac{6+\sqrt{21}}{5} - \frac{6-\sqrt{21}}{5}$$

$$= \frac{6+\sqrt{21}-6+\sqrt{21}}{5}$$

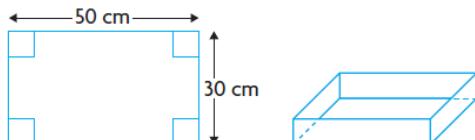
$$= \frac{2\sqrt{21}}{5} \text{ seconds}$$

$$\approx 1.83 \quad [\approx 2.11 - 0.28]$$

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(12) \pm \sqrt{(12)^2 - 4(-5)(-3)}}{2(-5)} \\ &= \frac{-12 \pm \sqrt{144 - 60}}{-10} \\ &= \frac{-12 \pm \sqrt{84}}{-10} \\ &= \frac{-12 \pm \sqrt{4\sqrt{21}}}{-10} \\ &= \frac{-12 \pm 2\sqrt{21}}{-10} \\ &= \frac{-2(6 \pm \sqrt{21})}{-10} \\ &= \frac{6 \pm \sqrt{21}}{5} \end{aligned}$$

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13. An open-topped box can be made from a sheet of aluminium measuring 50 cm by 30 cm by cutting congruent squares from the four corners and folding up the sides. Write a polynomial function to represent the volume of such a box. Determine the range of side lengths that are possible for each square that is cut out and removed that result in a volume greater than 4000 cm³.



$$V = x(50-2x)(30-2x)$$

$$4000 < x(1500 - 100x - 60x + 4x^2)$$

$$0 < 4x^3 - 160x^2 + 1500x - 4000$$

$$0 < 4(x^3 - 40x^2 + 375x - 1000)$$

$$P(x) = 4(x^3 - 40x^2 + 375x - 1000)$$

$$P(5) = 0$$

$$\begin{array}{r} 5 \mid 1 \ -40 \ 375 \ -1000 \\ \downarrow \quad 5 \ -175 \ 1000 \\ 1 \ -35 \ 200 \ \text{OR} \end{array}$$

$$\therefore P(x) = (x-5)(x^2 - 35x + 200)$$

$$x = \frac{-(-35) \pm \sqrt{(-35)^2 - 4(1)(200)}}{2(1)}$$

$$= \frac{35 \pm \sqrt{425}}{2}$$

$$= \frac{35 \pm 5\sqrt{17}}{2}$$

$$= \frac{35 \pm 5\sqrt{17}}{2}$$

$$\therefore V = 4000 \text{ cm}^3,$$

$$\text{if } x = 5 \text{ cm, } \frac{35-5\sqrt{17}}{2} = 7.19 \quad \text{and } x = \frac{35+5\sqrt{17}}{2} = 27.8$$

But, $0 < x < 15$
(because $30-2x > 0$ for width)

\therefore the range of lengths of x

for a volume greater than 4000 cm^3

$$\text{is } \left\{ x \in \mathbb{R} \mid 5 < x < \frac{35-5\sqrt{17}}{2} \right\}.$$

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Without a calculator, explain why

$-4x^{12} - 7x^6 + 9x^2 + 20 < 30 + 11x^2$ has a solution of $-\infty < x < \infty$.

$$-4x^{12} - 7x^6 + 9x^2 + 20 - 30 - 11x^2 < 0$$

$$-4x^{12} - 7x^6 - 2x^2 - 10 < 0$$

Regardless of the value of x chosen,
 x^{12} , x^6 , and x^2 will always result in a positive number.

Then multiply by the negative coefficients (-4, -7, -2)
will result in negative values.

You will then have 3 negative values and -10,
which when added will always be negative,
which is by definition < 0 .

\therefore the solution is $-\infty < x < \infty$.

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15. Explain why the following solution strategy fails, and then solve the inequality correctly.

Solve: $(x+1)(x-2) > (x+1)(-x+6)$.

Divide both sides by $x+1$ and get $x-2 > -x+6$.

Add x to both sides: $2x-2 > 6$.

Add 2 to both sides: $2x > 8$.

Divide both sides by 2: $x > 4$.

*Fails :: you are not allowed
to divide both sides by $x+1$.*

$$(x+1)(x-2) > (x+1)(-x+6)$$

$$x^2 - x - 2 > -x^2 + 5x + 6$$

$$x^2 + x^2 - x - 5x - 2 - 6 > 0$$

$$2x^2 - 6x - 8 > 0$$

$$2(x^2 - 3x - 4) > 0$$

$$2(x-4)(x+1) > 0$$

$$P(x) = 0 \text{ if } x = 4 \text{ or } x = -1$$

$x < -1$	$-1 < x < 4$	$x > 4$
$(-) (-)$	$(-) (+)$	$(+) (+)$
$= +$	$= -$	$= +$
Yes	No	Yes

$$\therefore \text{Solution: } (-\infty, -1) \cup (4, \infty)$$