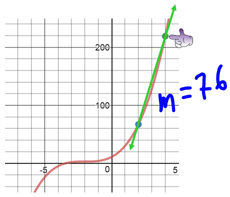
4.4 Rates of Change in Polynomial Functions

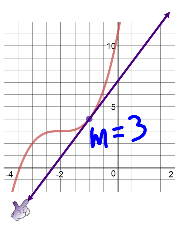
Math Learning Target:
"For any polynomial function, I can find the average rate of change and rate of change."

Ex. 1: Consider $y=(x+2)^3+3$
a) Find the average rate of change on the interval [2, 4]
b) Find the instantaneous rate of change at x = -1
c) Find the equation of the tangent line at x = -1

a) aroc
$= m_{secant}$
$= \frac{f(x_2)-f(x_1)}{x_2-x_1}$
$= \frac{[(4+2)^3+3]-[(2+2)^3+3]}{4-2}$
$= \frac{[216+3]-[64+3]}{2}$
$= \frac{152}{2}$
$= 76$
∴ on average, y increases by 76 units per unit of x, from x=2 to x=4

b) iroc = $m_{tangent}$
$= \frac{f(-1+h)-f(-1)}{h}$, as $h \to 0$
$= \frac{[(-1+h+2)^3+3]-[(-1+2)^3+3]}{h}$, as $h \to 0$
$= \frac{[(h+1)^3+3]-[(1)^3+3]}{h}$, as $h \to 0$
$= \frac{[h^3+3h^2+3h+1+3]-(4)}{h}$, as $h \to 0$
$= \frac{h^3+3h^2+3h}{h}$, as $h \to 0$
$= \frac{h(h^2+3h+3)}{h}$, as $h \to 0$
$= h^2+3h+3$, as $h \to 0$
$= 3$
∴ at x=-1, the y is increasing by 3 units per 1 unit of x.





c) eq'n of tangent at x=-1
$y=(-1+2)^3+3$
$= 1^3+3$
$= 4$
∴ (-1, 4)
∴ $y = mx+b$
$= 3x+b$
$4 = 3(-1)+b$
$4 = -3+b$
$4+3 = b$
$b = 7$
∴ $y=3x+7$ is the equation of the tangent at x=-1.

Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$(x+y)^3$

$x^3+3x^2y+3xy^2+y^3$

$x^4+4x^3y+6x^2y^2+4xy^3+y^4$

$(x+y)^5$

$x^5+5x^4y^1+\frac{5\cdot 4}{2}x^3y^2$

$x^5+5x^4y+10x^3y^2+\frac{10\cdot 3}{3}x^2y^3 \quad \frac{10\cdot 2}{4}xy^4$

$+10x^2y^3+5xy^4+y^5$

$(x+2)^5$

$=x^5+5x^4(2)+10x^3(2)^2+10x^2(2)^3+5x(2)^4+(2)^5$