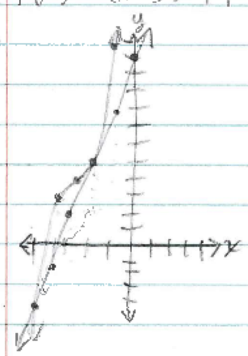
MHF 4UI | 4.4 Rates of Change Wkst

1. $f(x) = (x+3)^3 + 4$

a)



b) AROC $[-2, 1]$

$= m_{secant}$

$= \dfrac{f(-2) - f(1)}{-2-1}$

$= \dfrac{[(-2+3)^3+4] - [(1+3)^3+4]}{-3}$

$= \dfrac{[5] - [68]}{-3}$

$= 21$

$\therefore$ on average, y increases by 21 units per unit of x, from $x=-2$ to $x=1$

c) $iroc = m_{tangent}$ at $x=-1$

$= \dfrac{f(-1+h) - f(-1)}{h}$, as $h \to 0$

$= \dfrac{[(-1+h+3)^3+4] - [(-1+3)^3+4]}{h}$, $h \to 0$

$= \dfrac{[(h+2)^3+4] - [(2)^3+4]}{h}$, as $h \to 0$

$= \dfrac{[h^3 + 3h^2(2) + 3h(2)^2 + (2)^3 + 4] - (8+4)}{h}$, as $h \to 0$

$= \dfrac{h^3 + 6h^2 + 12h + 12 - 12}{h}$, as $h \to 0$

$= \dfrac{h(h^2 + 6h + 12)}{h}$, as $h \to 0$

$= h^2 + 6h + 12$, as $h \to 0$

$= 12$

$\therefore$ at $x=-1$, y is increasing by 12 units per 1 unit of x.

d) Find eq'n of tangent at $x=-1$

$f(-1) = (-1+3)^3 + 4$

$= 2^3 + 4$

$= 8 + 4$

$= 12$

$\because y = 12x + b$

$12 = 12(-1) + b$

$12 = -12 + b$

$24 = b$

$\therefore y = 12x + 24$ is the eq'n of the tangent at $x=-1$.

e) $f(x) = (x+3)^3 + 4 \ \cap \ y = 3x + 11$

$(x+3)^3 + 4 = 3x + 11$

$x^3 + 3x^2(3) + 3x(3)^2 + 1(3)^3 + 4 = 3x + 11$

$x^3 + 9x^2 + 27x + 27 + 4 - 3x - 11 = 0$

$x^3 + 9x^2 + 24x + 20 = 0$

$P(-2) = 0 \quad \therefore x+2$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & 9 & 24 & 20 \\ & \downarrow & -2 & -14 & -20 \\ \hline & 1 & 7 & 10 & 0R \end{array}$$

$\therefore (x+2)(x^2 + 7x + 10) = 0$

$(x+2)(x+2)(x+5) = 0$

$(x+2)^2(x+5) = 0$

$\therefore x = -2$ or $x = -5$

$\therefore (-2, 5) \quad (-5, -4)$

4.4 Rates of Change Wkst (p.2)

2. Given cubic graph

zeros: -3, 2

order: 1, 2

a) thru $(-2,-8)$

$f(x)=a(x+3)(x-2)^2$

$-8=a(-2+3)(-2-2)^2$

$-8=a(1)(-4)^2$

$-8=16a$

$a=-\frac{1}{2}$

$\therefore f(x)=-\frac{1}{2}(x+3)(x-2)^2$ is the equation

b) aroc $= m_{secant}$

$=\dfrac{f(4)-f(-2)}{4-(-2)}$

$=\dfrac{\left[-\frac{1}{2}(4+3)(4-2)^2\right]-\left[-\frac{1}{2}(-2+3)(-2-2)^2\right]}{6}$

$=\dfrac{\left[-\frac{1}{2}(7)(4)\right]-\left[-\frac{1}{2}(1)(16)\right]}{6}$

$=\dfrac{[-14]-[-8]}{6}$

$=\dfrac{-6}{6}$

$=-1$

$\therefore$ on average, y decreases by 1 unit per unit of x, from $x=-2$ to $x=4$.

c) iroc $= m_{tangent}$ (at $x=2$)

$=\dfrac{f(2+h)-f(2)}{h}$, as $h\to 0$

$=\dfrac{\left[-\frac{1}{2}(2+h+3)(2+h-2)^2\right]-\left[-\frac{1}{2}(2+3)(2-2)^2\right]}{h}$, as $h\to 0$

$=\dfrac{\left[-\frac{1}{2}(h+5)(h)^2\right]-\left[-\frac{1}{2}(5)(0)^2\right]}{h}$, as $h\to 0$

$=\dfrac{-\frac{1}{2}(h+5)h^2-0}{h}$, as $h\to 0$

$=-\frac{1}{2}h(h+5)$, as $h\to 0$

$=0$

d) $f(x)<-3$

$-\frac{1}{2}(x+3)(x-2)^2<-3$

$(x+3)(x-2)^2>6$

$(x+3)(x^2-4x+4)>6$

$x^3-4x^2+4x+3x^2-12x+12-6>0$

$x^3-x^2-8x+6>0$

$P(3)=3^3-3^2-8(3)+6$

$=0$ $\therefore x-3$ is a factor

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -8 & 6 \\ & \downarrow & 3 & 6 & -6 \\ \hline & 1 & 2 & -2 & 0R \end{array}$$

$\therefore (x-3)(x^2+2x-2)>0$

$\therefore x=3$ or x Now?

$x^2+2x-2=0$

$x^2+2x+1^2-1^2-2=0$

$(x+1)^2-3=0$

$(x+1)^2=3$

$x+1=\pm\sqrt{3}$

$x=-1\pm\sqrt{3}$

$\therefore x=-1-\sqrt{3}$ or $x=-1+\sqrt{3}$

$\doteq -2.73$ $\doteq 0.73$

$x<-1-\sqrt{3}$	$-1-\sqrt{3}<x<-1+\sqrt{3}$	$-1+\sqrt{3}<x<3$	$x>3$
$(-)(+)$	$(-)(-)$	$(-)(+)$	$(+)(+)$
$=-$	$=+$	$=-$	$=+$
No	Yes	No	Yes

$\therefore f(x)<-3$

$-1-\sqrt{3}<x<-1+\sqrt{3}$ or $x>3$

$(-2.73<x<0.73)$

$(x-3)(x^2+2x-2)>0$

$\downarrow$

$x=3$

$\hookrightarrow x^2+2x-2=0$

$$\underbrace{x^2+2x+1}-1-2=0$$

$(x+1)^2-3=0$

$(x+1)^2=3$

$x+1=\pm\sqrt{3}$

$x=-1\pm\sqrt{3}$

$x=-1+\sqrt{3}$ or $x=-1-\sqrt{3}$

$(x-3)(x^2+2x-2)>0$

$(x-3)(x-(-1+\sqrt{3}))(x-(-1-\sqrt{3}))>0$

$(x-3)(x+1-\sqrt{3})(x+1+\sqrt{3})>0$