5.4 Solving Rational Equations

Math Learning Target:

"I can state the restrictions on the variable in any rational equation. Then, I can solve the equation both algebraically and graphically. Finally, I can construct (and solve) a rational equation that arises from a real application."

Ex. 1: Solve $\{x \in \mathbb{R}\}$.

$$\frac{1}{3x}-\frac{1}{2}=\frac{5}{6x}$$

Restrictions: $x \neq 0$

$LCD = 6x$

$$\cancel{6x}^{2}\left(\frac{1}{\cancel{3x}}\right)-\cancel{6x}^{3x}\left(\frac{1}{\cancel{2}}\right)=\cancel{6x}^{1}\left(\frac{5}{\cancel{6x}}\right)$$

$2-3x=5$

$-3x=5-2$

$-3x=3$

$x=-1$

Warm-up

$$\frac{1}{3}x-\frac{1}{5}=\frac{x}{6}$$

$$\cancel{30}^{10}\left(\frac{1}{\cancel{3}}x\right)-\cancel{30}^{6}\left(\frac{1}{\cancel{5}}\right)=\cancel{30}^{5}\left(\frac{x}{\cancel{6}}\right)$$

$10(x)-6(1)=5(x)$

$10x-6=5x$

$10x-5x=6$

$5x=6$

$x=\frac{6}{5}$

Ex. 2

a) Determine a function whose zeros are the solutions to:

$$\frac{5}{4}=\frac{1}{x}-\frac{1}{x-5}$$

$$0=\frac{1}{x}-\frac{1}{x-5}-\frac{5}{4}$$

$$f(x)=\frac{1}{x}-\frac{1}{x-5}-\frac{5}{4}$$

$\times$ $LCD = 4x(x-5)$

$$=\frac{1}{x}\left(\frac{4(x-5)}{4(x-5)}\right)-\frac{1}{(x-5)}\left(\frac{4x}{4x}\right)-\frac{5}{4}\left(\frac{x(x-5)}{x(x-5)}\right)$$

$$=\frac{4x-20-(4x)-5(x^2-5x)}{4x(x-5)}$$

$$=\frac{4x-20-4x-5x^2+25x}{4x(x-5)}$$

$$=\frac{-5x^2+25x-20}{4x(x-5)}$$

b) Solve for the zeros algebraically $\{x \in \mathbb{R}\}$. Check your solution

$$\frac{5}{4}=\frac{1}{x}-\frac{1}{x-5}$$

$LCD = 4x(x-5)$

$$\cancel{4}x(x-5)\left(\frac{5}{\cancel{4}}\right)=4\cancel{x}(x-5)\left(\frac{1}{\cancel{x}}\right)-4x\cancel{(x-5)}\left(\frac{1}{\cancel{x-5}}\right)$$

$$5x(x-5)=4(x-5)-4x$$

$$5x^2-25x=4x-20-4x$$

$$5x^2-25x+20=0$$

$$5(x^2-5x+4)=0$$

$$5(x-1)(x-4)=0$$

$\therefore x=1$ or $x=4$

Check: $x=1$

$LS=\frac{5}{4}$ $RS=\frac{1}{x}-\frac{1}{x-5}$

$=\frac{1}{1}-\frac{1}{1-5}$

$=1-\frac{1}{-4}$

$=1+\frac{1}{4}$

$=\frac{5}{4}$

$\therefore LS=RS$

$\therefore x=1$ is a root.

$x=4$

$LS=\frac{5}{4}$ $RS=\frac{1}{x}-\frac{1}{x-5}$

$=\frac{1}{4}-\frac{1}{4-5}$

$=\frac{1}{4}-\frac{1}{-1}$

$=\frac{1}{4}+1$

$=\frac{5}{4}$

$\therefore LS=RS$

$\therefore x=4$ is a root.

Entertainment:
pp. 285-287 #3b, 4b (do not "verify"), 5c, 6abc, **9, 11 (see Example 4, text), **12
Challenge: #16 (use desmos).
Answer for #16a) should be: at 0.417 sec and 1.705 sec.

Legend:
** final answers must be stated as simplified exact values (not rounded!)

$$5(x^2-5x+4)=0$$

$$x^2-5x+4=0$$

$$\underbrace{x^2-5x+\left(\frac{5}{2}\right)^2}-\left(\frac{5}{2}\right)^2+4=0$$

$$\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+\frac{16}{4}=0$$

$$\left(x-\frac{5}{2}\right)^2-\frac{9}{4}=0$$

$$\left(x-\frac{5}{2}\right)^2=\frac{9}{4}$$

$$x-\frac{5}{2}=\pm\sqrt{\frac{9}{4}}$$

$$x-\frac{5}{2}=\pm\frac{3}{2}$$

$$x=\frac{5}{2}\pm\frac{3}{2}$$

$$x=\frac{5+3}{2} \quad \text{or} \quad x=\frac{5-3}{2}$$

$$=\frac{8}{2} \qquad\qquad =\frac{2}{2}$$

$$=4 \qquad\qquad =1$$

d f

Do: p. 272 #1, 5ad, 6, 8*, 9, 10**.
Enrich Yourself!... p. 274 #12, 13, 14***

Answers that need to be corrected in the text:

8* f(x) has a VA at x=1; g(x) has a HA at y=0.5.
Also, f(x) has a HA at y=3; g(x) has a VA at x=-1.5

10** The concentration increases over the 24 h period and approaches approx. 1.85 mg/L

14***a) f(x) and m(x)
b) g(x)

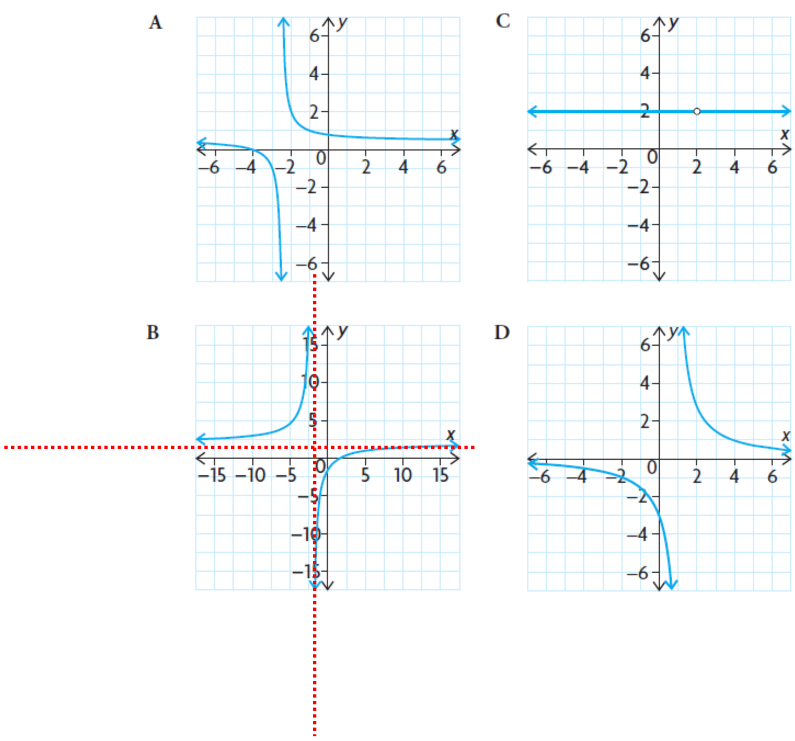
p. 272 #1d

1. Match each function with its graph.

a) $h(x) = \dfrac{x+4}{2x+5}$ c) $f(x) = \dfrac{3}{x-1}$

b) $m(x) = \dfrac{2x-4}{x-2}$ d) $g(x) = \dfrac{2x-3}{x+2}$

VA : x = -2
HA : y = 2



p. 274 #9f

9. The function $I(t) = \dfrac{15t + 25}{t}$ gives the value of an investment, in thousands of dollars, over t years.

a) What is the value of the investment after 2 years?

b) What is the value of the investment after 1 year?

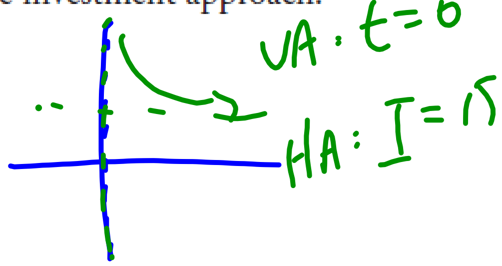
c) What is the value of the investment after 6 months?

d) There is an asymptote on the graph of the function at $t = 0$. Does this make sense? Explain why or why not.

e) Choose a very small value of t (a value near zero). Calculate the value of the investment at this time. Do you think that the function is accurate at this time? Why or why not?

f) As time passes, what will the value of the investment approach?

↳ as $t \to \infty$, $h \to \$15\,000$



p. 274 #10

10. An amount of chlorine is added to a swimming pool that contains pure water. The concentration of chlorine, c, in the pool at t hours is given by $c(t) = \frac{2t}{2+t}$, where c is measured in milligrams per litre. What happens to the concentration of chlorine in the pool during the 24 h period after the chlorine is added?

$t=24$

$$c(24) = \frac{2(24)}{2+24}$$

$$= \frac{48}{26}$$

$$\doteq 1.846 \text{ mg/L}$$