5.6 Rates of Change in Rational Functions

Math Learning Target:

"I can find the exact average rate of change and (instantaneous) rate of change of any rational function."

Recall: Simplify: $\frac{h-5}{h-2} - \frac{-5}{-2}$

LCD $= 2(h-2)$

$= \frac{h-5}{h-2} - \frac{5}{2}$

$= \frac{2(h-5)}{2(h-2)} - \frac{5(h-2)}{2(h-2)}$

$= \frac{2h-10-5h+10}{2(h-2)}$

$\rightarrow \frac{-3h}{2(h-2)}$

Restriction $h \neq 2$

Ex. 1: a) Find the slope of the tangent to $f(x) = \frac{x}{x+3}$ at $x = -5$.

b) Why can there not be a tangent line at $x = -3$?

iroc = $m_{tangent}$ $x=-5$

$= \frac{f(-5+h)-f(-5)}{h}$, as $h \to 0$

$= \frac{\frac{-5+h}{-5+h+3} - \left(\frac{-5}{-5+3}\right)}{h}$, as $h \to 0$

$= \left[\frac{-5+h}{h-2} - \left(\frac{-5}{-2}\right)\right] \div h$, as $h \to 0$

$= \left[\frac{h-5}{h-2} - \frac{5}{2}\right] \div h$, as $h \to 0$

$= \left[\frac{2(h-5)-5(h-2)}{2(h-2)}\right] \div h$, as $h \to 0$

$= \left[\frac{2h-10-5h+10}{2(h-2)}\right] \times \frac{1}{h}$, as $h \to 0$

$= \frac{-3\cancel{h}}{2(h-2)} \times \frac{1}{\cancel{h}}$, as $h \to 0$

$= \frac{-3}{2(h-2)}$, as $h \to 0$

$= \frac{-3}{2(0-2)}$

$= \frac{-3}{-4}$

$= \frac{3}{4}$

$\therefore$ the slope of the tangent at $x=-5$ is $\frac{3}{4}$

ENTERTAINMENT: With a partner(s), complete the worksheet.