MHF 4UI Name: ______________________

Rates of Change in Rational Functions: Worksheet

1. Consider the function defined by $f(x)=\dfrac{x}{x+2}$.

   a) Graph the function.

   b) Find the average rate of change of the function on the interval [2.99 , 3].
   Repeat for the interval [2.999 , 3].
   What is a reasonable estimate for the rate of change at $x = 3$?

   c) Use the algebraically simplified difference quotient
   to determine the **exact** rate of change of $f(x)$ at $x = 3$.

   d) Determine the equation of the tangent line to $f(x)$ at $x = 3$.
   Express your answer in $y = mx + b$ form.

   e) Solve $f(x) \leq 2$ algebraically. Verify graphically using part a).

$f(x)=\dfrac{x}{x+2}$

b) aroc = m_{secant} [2.99, 3]

$$= \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$= \frac{\frac{3}{3+2}-\frac{2.99}{2.99+2}}{3-2.99}$$

$$= \frac{\frac{3}{5}-\frac{2.99}{4.99}}{0.01}$$

$$\doteq 0.08016$$

[2.999, 3]

$$\text{aroc} = \frac{\frac{3}{3+2}-\frac{2.999}{2.999+2}}{3-2.999}$$

$$= \frac{\frac{3}{5}-\frac{2.999}{4.999}}{0.001}$$

$$\doteq 0.080016$$

$\therefore$ a reasonable estimate at $x=3$ is y is increasing at a rate of 0.08 units for every 1 unit of x.

1. Consider the function defined by $f(x)=\frac{x}{x+2}$.

c) Use the algebraically simplified difference quotient to determine the **exact** rate of change of $f(x)$ at $x = 3$.

c) $\text{roc} = \frac{f(x+h)-f(x)}{h}$, as $h \to 0$ (at $x=3$)

$\frac{x}{x+2}$

$= \frac{\frac{3+h}{3+h+2} - \frac{3}{3+2}}{h}$, as $h \to 0$

$= \left(\frac{3+h}{h+5} - \frac{3}{5}\right) \div h$, as $h \to 0$

$= \left(\frac{3+h}{h+5}\cdot\frac{5}{5} - \frac{3}{5}\left(\frac{h+5}{h+5}\right)\right) \times \frac{1}{h}$, as $h \to 0$

$= \left(\frac{15+5h-3h-15}{5(h+5)}\right) \times \frac{1}{h}$, as $h \to 0$

$= \frac{2\cancel{h}}{5(h+5)} \times \frac{1}{\cancel{h}}$, as $h \to 0$

$= \frac{2}{5(h+5)}$ as $h \to 0$

$= \frac{2}{5(0+5)}$

$= \frac{2}{5(5)}$

$= \frac{2}{25}$

$= 0.08$

$\therefore$ the exact rate of change at $x=3$ is y is increasing at a rate of 0.08 units per 1 unit of x.

d) Determine the equation of the tangent line to $f(x)$ at $x = 3$. Express your answer in $y = mx + b$ form.

d) at $x=3$, $m = 0.08$ (from c)

$f(3) = \frac{3}{3+2}$

$= \frac{3}{5}$

$\therefore$ to find the eq'n of the tangent

$y = 0.08x + b$

$\frac{3}{5} = \frac{2}{25}(3) + b$

$\frac{3}{5} = \frac{6}{25} + b$

$\frac{15}{25} - \frac{6}{25} = b$

$\therefore b = \frac{9}{25}$

$\therefore y = \frac{2}{25}x + \frac{9}{25}$ is the equation of the tangent at $x=3$.

(or $y = 0.08x + 0.36$)

1. Consider the function defined by $f(x)=\dfrac{x}{x+2}$.

e) Solve $f(x)\leq 2$ algebraically. Verify graphically using part a).

$$\frac{x}{x+2}\leq 2$$

$$\frac{x}{x+2}-2\leq 0$$

$$\frac{x}{x+2}-2\left(\frac{x+2}{x+2}\right)\leq 0$$

$$\frac{x-2x-4}{x+2}\leq 0$$

$$\frac{-x-4}{x+2}\leq 0$$

$$\frac{-(x+4)}{x+2}\leq 0$$

$$\frac{x+4}{x+2}\geq 0$$

Interval boundaries

$x=-2,\ x=-4$

	$x<-4$	$-4<x<-2$	$x>-2$
	$\frac{(-)}{(-)}$ $=+$	$\frac{(+)}{(-)}$ $=-$	$\frac{(+)}{(+)}$ $=+$
$f(x)>0$	Yes	No	Yes

$\therefore (-\infty,-4]\cup[-2,\infty)$