

MHF 4UI

Name: _____

Unit 5: Rational Functions, Equations and Inequalities
“Creating a Model”

A **mathematical model** is a mathematical representation of the essential aspects of a system. These models can be expressed in many forms, such as equations, formulas, inequalities, etc...

Models can be very simple. For example, a model of the volume of a rectangular prism is:

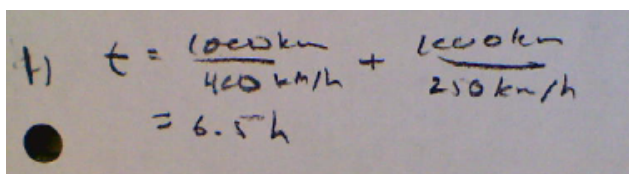
$$\text{volume} = \text{length} \times \text{width} \times \text{height}$$

We will create, and solve, some slightly more complex models today.

In groups of 3 or 4...For each scenario, create a mathematical model in the form of a **rational equation**. Define the variable(s) introduced. Then solve the equation.

1. An aircraft maintains an average airspeed of 325 km/h (when there is no wind) between two airports on both the outbound and return trips. On one trip, on the outbound trip there is an average tailwind of 75 km/h and on the return trip a headwind of the same speed. If the airports are 1000 km apart, how long does it take to fly the round trip? [Answer: 6.5 h]

Recall: average speed = distance \div time



Handwritten calculation showing the time taken for a round trip between two airports 1000 km apart. The aircraft has an average airspeed of 325 km/h. On the outbound trip, there is a tailwind of 75 km/h, resulting in a ground speed of 400 km/h. On the return trip, there is a headwind of 75 km/h, resulting in a ground speed of 250 km/h. The total time is calculated as the sum of the time for each leg: $t = \frac{1000 \text{ km}}{400 \text{ km/h}} + \frac{1000 \text{ km}}{250 \text{ km/h}} = 6.5 \text{ h}$.

2. Dieter makes a journey of 430 km, travelling 160 km by bus, and the rest by car. If the car averages 10 km/h faster than the bus, and the entire journey is 5 h, what is the speed of the car?
[Answer: 90 km/h]

Let v represent the speed of the car, in km/h.

1. v = speed of car

2) $S = \frac{160}{v-10} + \frac{270}{v} \rightarrow 5v(v-10) = \frac{160v(v-10)}{v-10} + \frac{270v(v-10)}{v}$

$$5v^2 - 50v - 160v - 270v + 2700 = 0$$

$$5(v^2 - 96v + 540) = 0$$

$$5(v-6)(v-90) = 0$$

$v = 6$ or $v = 90$
inadmissible, v must be > 10

$$\frac{160}{v} + \frac{270}{v+10}$$

$$\frac{d}{s} = t$$

$$d = st$$

$$s = \frac{d}{t} \quad t = \frac{d}{s}$$

3. The sport biathlon combines cross-country skiing with rifle shooting. At the Olympics, competitors start at singly at one-minute intervals. Frank, who will start immediately after Anatole, feels that no matter how fast Anatole is, he can ski an average of 1 km/h faster. What is the average speed of each biathlete when they ski, if Frank overtakes Anatole at the first shooting range, which is 4 km from the start? [Answer: Anatole 15 km/h and Frank 16 km/h]

$$\frac{1}{60} = \frac{4}{v} - \frac{4}{v+1}$$

$$\frac{60v(v+1)}{60} = \frac{4(60v)(v+1)}{v} - \frac{4(60v)(v+1)}{v+1}$$

$$v^2 + v = 240v + 240 - 240v$$

$$v^2 + v - 240 = 0$$

$$(v-15)(v+16) = 0$$

$$v = 15 \text{ km/h}$$

4. Students sent flowers costing \$20 to a sick classmate. There were four fewer students contributing than was planned, requiring each of the others to give 25 cents more. How many students contributed to the gift? [Answer: 16]

$$\begin{aligned}
 4. \quad p &= \frac{20}{n} \quad \text{where } p \text{ is \$ per student} \\
 &\quad n = \# \text{ students} \\
 p + 0.25 &= \frac{20}{n-4} \\
 \frac{20}{n} + 0.25 &= \frac{20}{n-4} \\
 \frac{20}{n-4} &= \frac{20}{n} + 0.25 \\
 20n &= 20(n-4) + 0.25n(n-4) \\
 20n &= 20n - 80 + 0.25n^2 - n \\
 0 &= 0.25n^2 - n - 80 \\
 0 &= 0.25(n^2 - 4n - 320) \\
 0 &= 0.25(n-20)(n+16) \\
 n &= 20 \quad \text{or} \quad n = -16
 \end{aligned}$$

5. Brendan buys a block of shares of the company *HuskyTech* for \$1875. When the share price increases by \$4 per share, he sells all but 15 of them for \$1740. How many shares did he buy? [Answer: 75]

$$\begin{aligned}
 &\text{let } p \text{ be price per share and } n \text{ be \# shares} \\
 5. \quad 14 &= \frac{1740}{n-15} - \frac{1875}{n} \\
 4n(n-15) &= 1740n - 1875(n-15) \\
 4n^2 - 60n &= 1740n - 1875n + 28125 \\
 4n^2 + 75n &= 28125 \\
 n &= 93.75, \quad n = 75 \\
 &\text{need an integer}
 \end{aligned}$$