6.4 Transformations of Trigonometric Functions

Math Learning Target:

"I can identify and graph any transformations of the trigonometric functions studied."

Ex.1: Graph for one period: $f(x) = -2\cos\left(\frac{1}{2}x - \frac{\pi}{2}\right) + 1$

RST

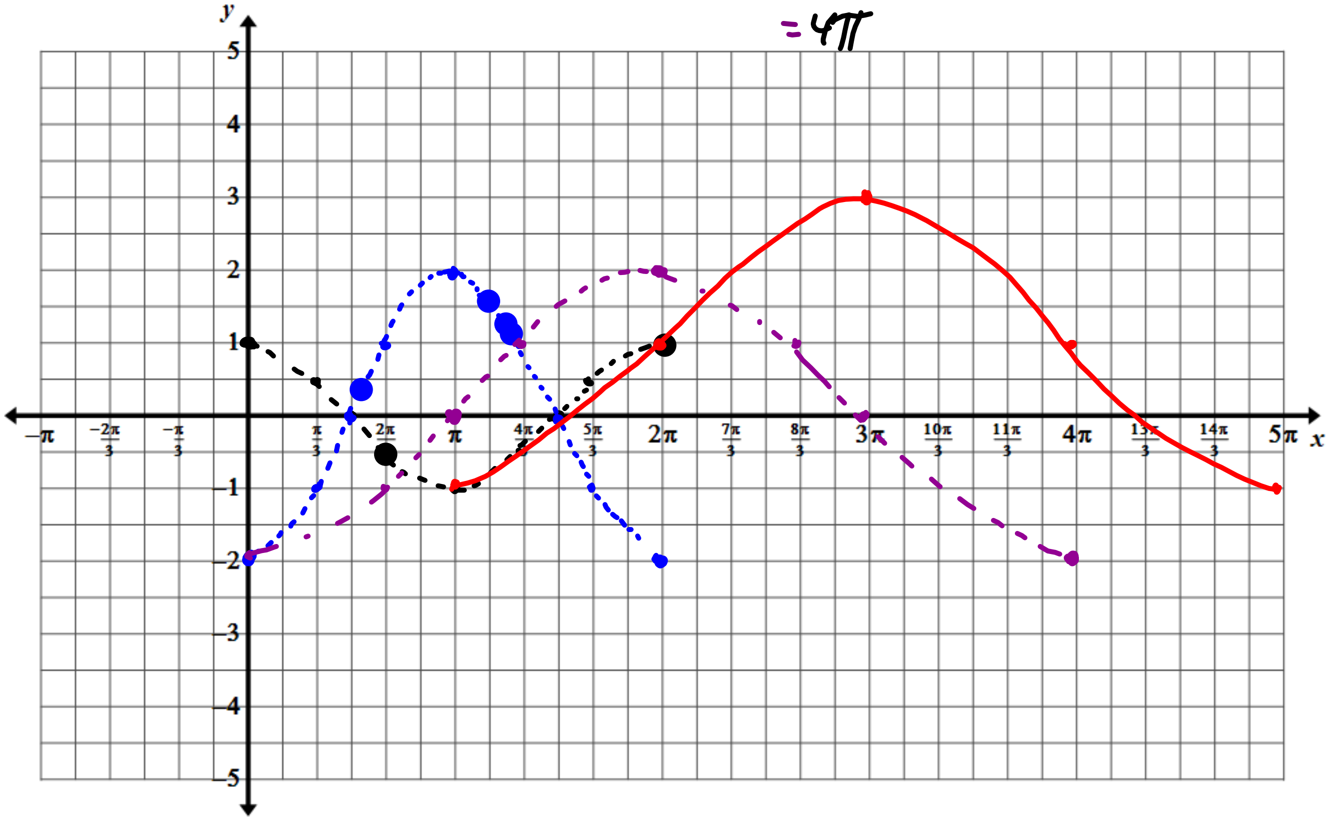
period $= \frac{2\pi}{k}$

$k = \frac{2\pi}{\text{period}}$

$= -2\cos\left(\frac{1}{2}(x-\pi)\right)+1$

period $= \frac{2\pi}{\frac{1}{2}}$

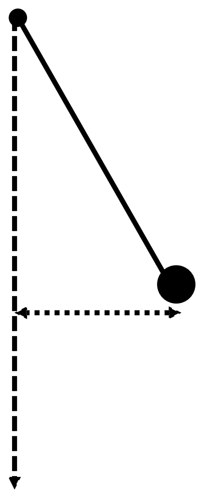
$= 4\pi$



Ex. 2: A large pendulum swings back and forth with a maximum of a 3 *m* horizontal distance from its centre, and completes one cycle every 4 seconds.

a) Graph this motion for two cycles, beginning it at the end of its swing.

b) Write one equation to model the horizontal distance from its centre, *d*, over time, *t*.



$a = 3$

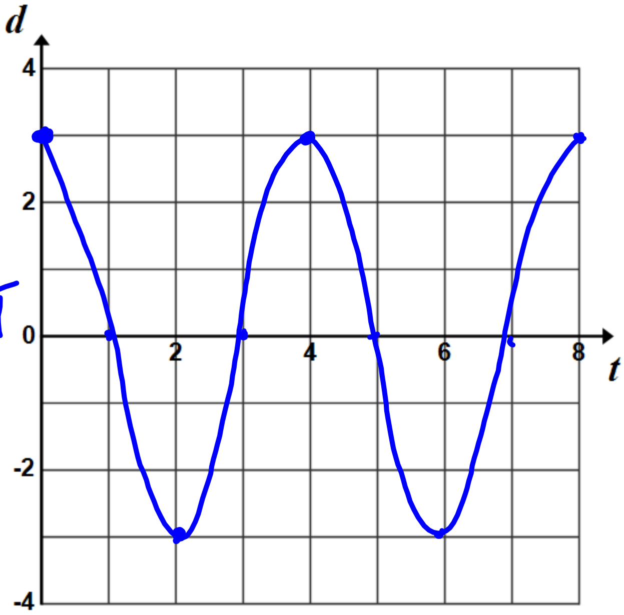
$c = 0$

$d = 0$

$k = \frac{2\pi}{\text{period}}$

$= \frac{2\pi}{4}$

$= \frac{\pi}{2}$



$y = a\cos(k(x-d)) + c$

$d = 3\cos\left(\frac{\pi}{2}(t-0)\right) + 0$

$d = 3\cos\left(\frac{\pi}{2}t\right)$

Entertainment: pp.343-346 #**1ad, 4bc, 5ac, 6c, 7bc, 8c*d*** graph (do not sketch),

9, 10* graph (do not sketch), **11, 12, 14a**

Entertainment: pp.343-346 #**1ad, 4bc, 5ac, 6c, 7bc, 8c*d*** graph (do not sketch),
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p. 8cd, 10

p.320 11, 12, 5b, 13

p.330 2ab

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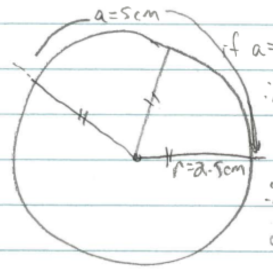
p.320 11, 12, 5b, 13

p.321 **5.** **a)** Determine the measure of the central angle that is formed by an arc length of 5 cm in a circle with a radius of 2.5 cm. Express the measure in both radians and degrees, correct to one decimal place.

b) Determine the arc length of the circle in part a) if the central angle is 200°.

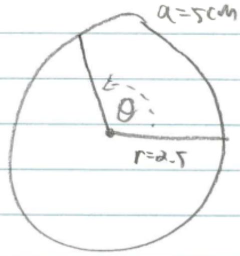
5 a) arc length = 5 cm
radius = 2.5 cm
find θ (central angle)



if a = 2.5 cm
$\therefore \theta = 1$ radian

$\therefore \theta = 2$ radians
or $\theta \doteq 114.59$
$\doteq 114.6°$

or a)



$a = \frac{\theta}{360°}$ (circumference)
$= \frac{\theta}{360°}(2\pi r)$
$5 = \frac{\theta}{360}(2\pi(2.5))$
$\therefore \theta = \frac{360(5)}{2\pi(2.5)}$
$= \frac{360°}{\pi}$
$\doteq 114.59$
$\doteq 114.6°$

or $\theta = \frac{360}{\pi} \times \frac{\pi}{180°}$
$= 2$ radians

b) if $\theta = 200°$
$a = \left(\frac{\theta}{360°}\right) 2\pi r$
$= \frac{200}{360}\left(2\pi(2.5)\right)$
$= \frac{5}{9}\left(2\pi(2.5)\right)$
$= \frac{25\pi}{9}$ cm

p.322

11. A wind turbine has three blades, each measuring 3 m from centre to tip. At a particular time, the turbine is rotating four times a minute.

a) Determine the angular velocity of the turbine in radians/second.

b) How far has the tip of a blade travelled after 5 min?

11. $r = 3m$ 4 rev/min.

a) ang. vel. $= \frac{\theta}{t}$

$\theta = 2\pi \times 4$
$= 8\pi$ rad.

$= \frac{8\pi \text{ rad}}{1 \text{ min}}$
$= \frac{8\pi}{1\text{min}} \times \frac{1\text{min}}{60\text{sec}}$
$= \frac{4\pi}{30} \frac{\text{rad}}{\text{sec}}$
$= \frac{2\pi}{15}$ rad/sec
$\doteq 0.418$ rad/sec

b) 1 rev. $= 2\pi r$
$= 2\pi(3)$
$= 6\pi$
4 rev/min $= 24\pi$ /min
$\therefore$ after 5min $= 120\pi$ m
$\doteq 376.99$
$\doteq 377$ m

p.322

12. A wheel is rotating at an angular velocity of 1.2π radians/s, while a point on the circumference of the wheel travels 9.6π m in 10 s.

a) How many revolutions does the wheel make in 1 min?

b) What is the radius of the wheel?

12. ang. vel. (ω) = 1.2π rad/sec

$\theta = 9.6\pi \text{ m} / 10 \text{ sec}$

a) $\omega = \frac{1.2\pi \text{ rad}}{\text{sec}} \times \frac{60 \text{ sec}}{1 \text{ min}}$

$= \frac{72\pi \text{ rad}}{\text{min}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}}$

$= 36 \text{ rev./min}$

b) $\theta = \frac{9.6\pi \text{ m}}{10 \text{ sec}} \times \frac{60 \text{ sec}}{1 \text{ min}}$

$= 57.6\pi \text{ m/min}$

$\therefore \frac{57.6\pi \text{ m}}{\text{min}} \times \frac{1 \text{ min}}{36 \text{ rev}}$

$= 1.6\pi \text{ m/rev.}$

$C = 2\pi r$

$1.6\pi \text{ m} = 2\pi r$

$\therefore r = \frac{1.6\pi \text{ m}}{2\pi}$

$= 0.8 \text{ m}$

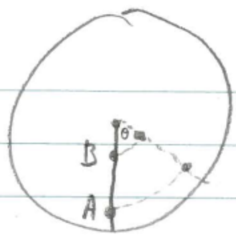
13. **T** Two pieces of mud are stuck to the spoke of a bicycle wheel. Piece A is closer to the circumference of the tire, while piece B is closer to the centre of the wheel.

a) Is the angular velocity at which piece A is travelling greater than, less than, or equal to the angular velocity at which piece B is travelling?

b) Is the velocity at which piece A is travelling greater than, less than, or equal to the velocity at which piece B is travelling?

c) If the angular velocity of the bicycle wheel increased, would the velocity at which piece A is travelling as a percent of the velocity at which piece B is travelling increase, decrease, or stay the same?

13.



a) $\omega = \frac{\theta}{t}$

in both cases, $\theta = 1$ radian,

∴ the angular velocity of each piece of mud is equal.

b) $v = \frac{d}{t}$, in this case, the distance piece A travels is greater than piece B, so velocity A > velocity B

c) $v_A = \frac{2\pi r_A}{t}$ $v_B = \frac{2\pi r_B}{t}$ % $\frac{v_A}{v_B} = \frac{2\pi r_A}{t} \div \frac{2\pi r_B}{t}$

$= \frac{2\pi r_A}{t} \times \frac{t}{2\pi r_B}$

$= \frac{r_A}{r_B}$ ∴ constant (i.e.) independent of ang. velocity p.330 5b, 13

5. Determine the exact value of each trigonometric ratio.

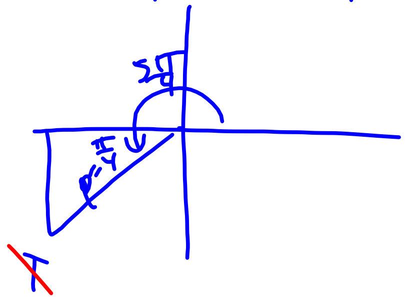
1   b) $\cos \frac{5\pi}{4}$

p.330 ans

$= -\cos\left(\frac{\pi}{4}\right)$

$= -\frac{\sqrt{2}}{2}$

$\theta = \frac{5\pi}{4}$ $\beta = \frac{\pi}{4}$ ← rad

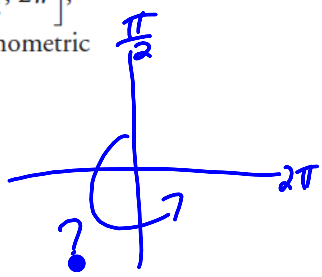


$\frac{5\pi}{3}$

$= \frac{5(\overset{60}{180})}{3}$

$= 300°$

12. If you are given an angle, θ, that lies in the interval $\theta \in \left[\frac{\pi}{2}, 2\pi\right]$, how would you determine the values of the primary trigonometric ratios for this angle?



13. You are given $\cos \theta = -\frac{5}{13}$, where $0 \leq \theta \leq 2\pi$.

a) In which quadrant(s) could the terminal arm of θ lie?

b) Determine all the possible trigonometric ratios for θ.

c) State all the possible radian values of θ, to the nearest hundredth.

13. $\cos\theta = \frac{-5}{13}$, $0 \leq \theta \leq 2\pi$

a) from CAST Rule either QII or QIII

b) $\cos\theta = \frac{x}{r}$

$\therefore x = -5,\ r = 13$

$y^2 = r^2 - x^2$

$= 13^2 - (-5)^2$

$= 144$

$r = \pm 12$

$\therefore \sin\theta = \frac{12}{13}$ or $\sin\theta = \frac{-12}{13}$

$\tan\theta = \frac{12}{-5}$ or $\tan\theta = \frac{-12}{-5} = \frac{12}{5}$

$\csc\theta = \frac{13}{12}$ or $\csc\theta = \frac{-13}{12}$

$\sec\theta = -\frac{13}{5}$

$\cot\theta = \frac{-5}{12}$ or $\cot\theta = \frac{5}{12}$

c) $\cos\beta = \frac{5}{13}$ (rad)

$\beta = \frac{5}{13}$

$\doteq 1.176$

$\therefore$ in QII, $\theta = \pi - \beta$

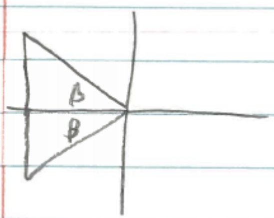
$\doteq 1.965$

$\doteq 1.97$ rad

in QIII, $\theta = \pi + \beta$

$\doteq 4.317$

$\doteq 4.32$ rad



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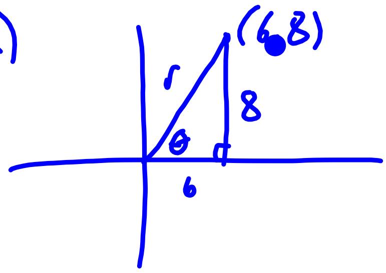
2. Each of the following points lies on the terminal arm of an angle in standard position.
   i) Sketch each angle.
   ii) Determine the value of r.
   iii) Determine the primary trigonometric ratios for the angle.
   iv) Calculate the radian value of θ, to the nearest hundredth, where $0 \le \theta \le 2\pi$.

   a) $(6, 8)$ c) $(4, -3)$
   b) $(-12, -5)$ d) $(0, 5)$

a)



$r^2 = 6^2 + 8^2$

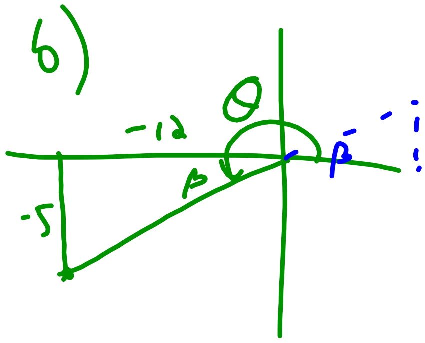
$r = 10$ units

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{8}{10} = \frac{4}{5}$ $\cos\theta = \frac{6}{10} = \frac{3}{5}$ $\tan\theta = \frac{8}{6} = \frac{4}{3}$

$\theta = \sin^{-1}\left(\frac{4}{5}\right) \doteq 0.927 \doteq 0.93$ $\theta = \cos^{-1}\left(\frac{3}{5}\right) \doteq 0.927$

b)



$\sin\theta = \frac{y}{r} = \frac{-5}{13}$ $\cos\theta = \frac{x}{r} = \frac{-12}{13}$ $\tan\theta = \frac{y}{x} = \frac{-5}{-12} = \frac{5}{12}$

$r^2 = (-12)^2 + (-5)^2$

$r = 13$

$\theta = \pi + \beta \doteq 3.536 \doteq 3.54$

$\beta \left\{ \begin{array}{l} \theta = \tan^{-1}\left(\frac{5}{12}\right) \\ \theta \doteq 0.394 \doteq 0.39 \end{array} \right.$

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5. Find an expression that describes the location of each of the following values for $y = \tan\theta$, where $n \in I$ and θ is in radians.

a) θ-intercepts b) vertical asymptotes

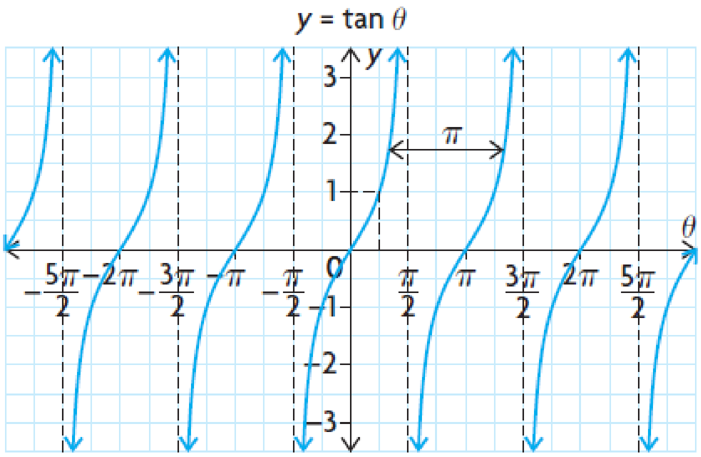
$\ldots -2\pi, -\pi, 0, \pi, 2\pi, \ldots$

$\{\theta \in \mathbb{R} \mid \theta = n\pi, n \in I \ (\mathbb{Z})\}$

$-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$\frac{(2n+1)\pi}{2}, n \in I$

$\{\theta \in \mathbb{R} \mid \theta = \frac{(2n+1)\pi}{2}, n \in I\}$



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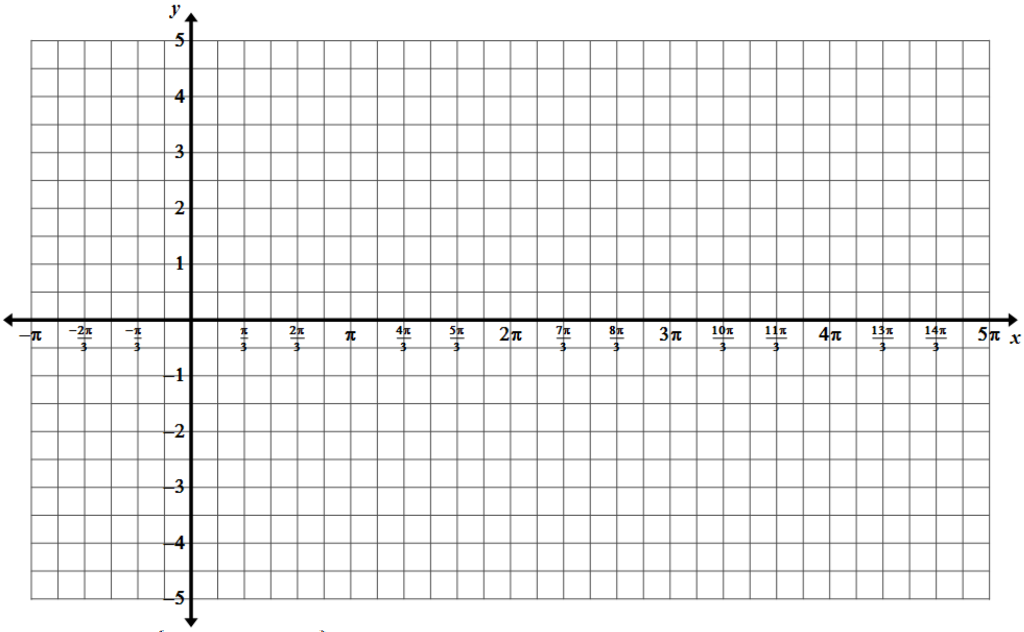
8. Sketch each graph for $0 \le x \le 2\pi$. Verify your sketch using graphing technology.

c) $y = -2\sin\left(2\left(x + \frac{\pi}{4}\right)\right) + 2$

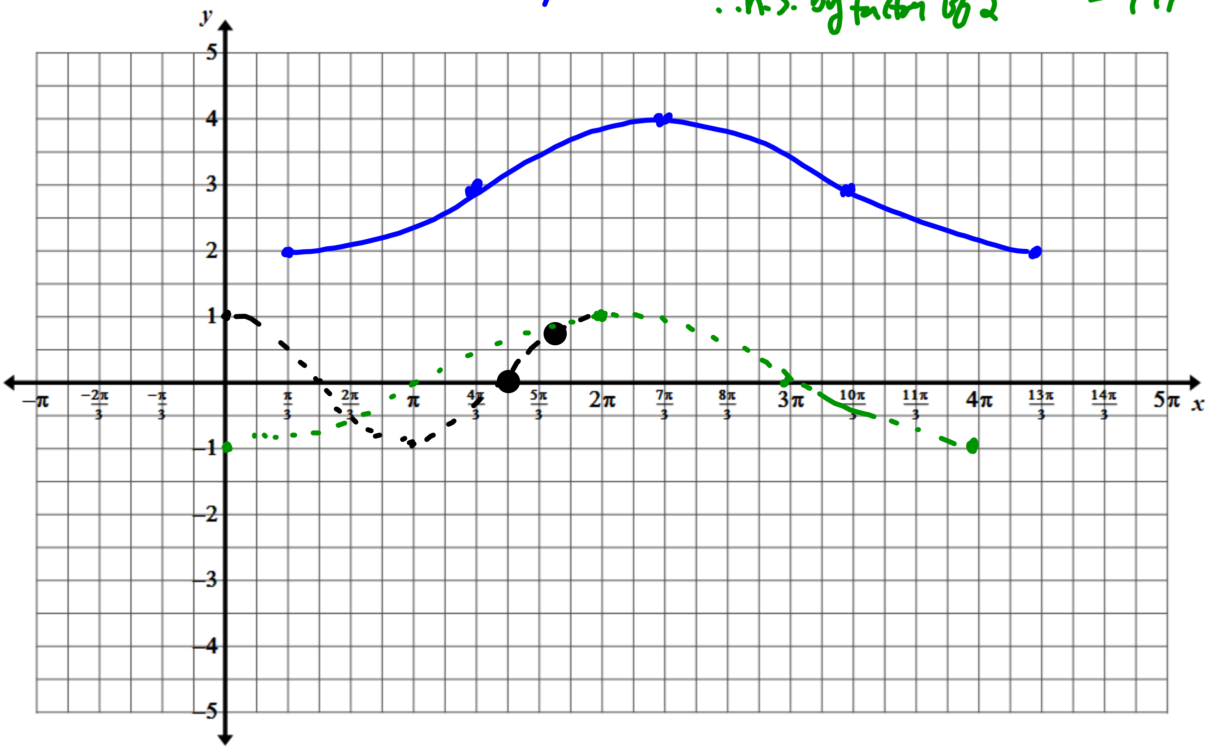


d) $y = -\cos\left(0.5x - \frac{\pi}{6}\right) + 3$

$= -\cos\left(0.5\left(x - \frac{\pi}{3}\right)\right) + 3$

$b = \frac{1}{2}$ $\therefore$ period $= \frac{2\pi}{\frac{1}{2}}$

$\therefore$ h.s. by factor of 2 $= 4\pi$



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10. A pendulum swings back and forth 10 times in 8 s. It swings through a total horizontal distance of 40 cm.

a) Sketch a graph of this motion for two cycles, beginning with the pendulum at the end of its swing.

b) Describe the transformations necessary to transform $y = \sin x$ into the function you graphed in part a).

c) Write the equation that models this situation.