WARM UP: please attempt this question on lined paper

$y = a(b^x)$

In the year 2025 the world's human population had reached approx. 8.1 billion (that's **8 100 000 000**). At this point, severe droughts brought on by climate change began to decimate agricultural crops causing the human population to decrease exponentially by approx. 32% per year.

a) write an equation to model the human population, y, in x years after 2025

$y = 8\,100\,000\,000(0.68)^x$

$b = 100\% \pm r$
$= 100\% - 32\%$
$= 68\%$
$= 0.68$

b) If this is the year 2060, how many people remain on Earth?

$\therefore x = 35 \quad (2060 - 2025)$

$y = 8\,100\,000\,000(0.68)^{35}$

$\doteq 11\,124.9$

$\approx 11\,124$ people

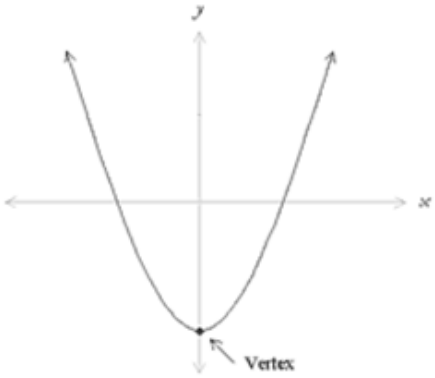
Cycle 4 Day 3: **Maximum/Minimum Values of Quadratic Relations**

Learning Goal(s): "I can identify when a parabola has a maximum or minimum value.
"I can find the maximum/minimum value for a quadratic relation and when it occurs.

We have observed that the graphs of quadratic relations are in the shape of a parabola.

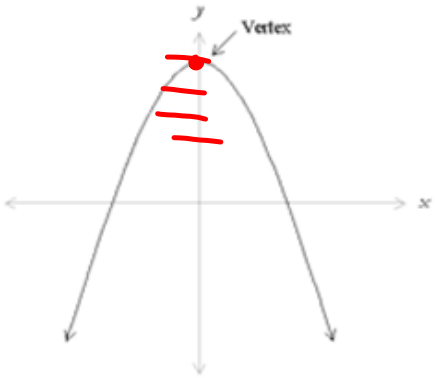
Some parabolas open up and their vertex would represent the minimum (lowest point) of the relation:

Opens Upward



Some parabolas open down and their vertex would represent the maximum (highest point) of the relation:

Opens Downward



If the equation of the quadratic is in vertex form finding the max/min value is easy since the equation tells the coordinates of the vertex.

$$y = a(x - h)^2 + k$$

If 'a' is positive, the parabola opens upwards and has a minimim value.

If 'a' is negative, the parabola opens downwards and has a maximum value.

The maximum/minimum value is k and it occurs when x = h

Ex.1 Given the quadratic relation $y = -3(x - 7)^2 + 13$

a) Does this relation have a maximum or a minimum value? maximum
How do you know? $a = -3$ (negative) → opens downward

b) What is the maximum value and when does it occur?

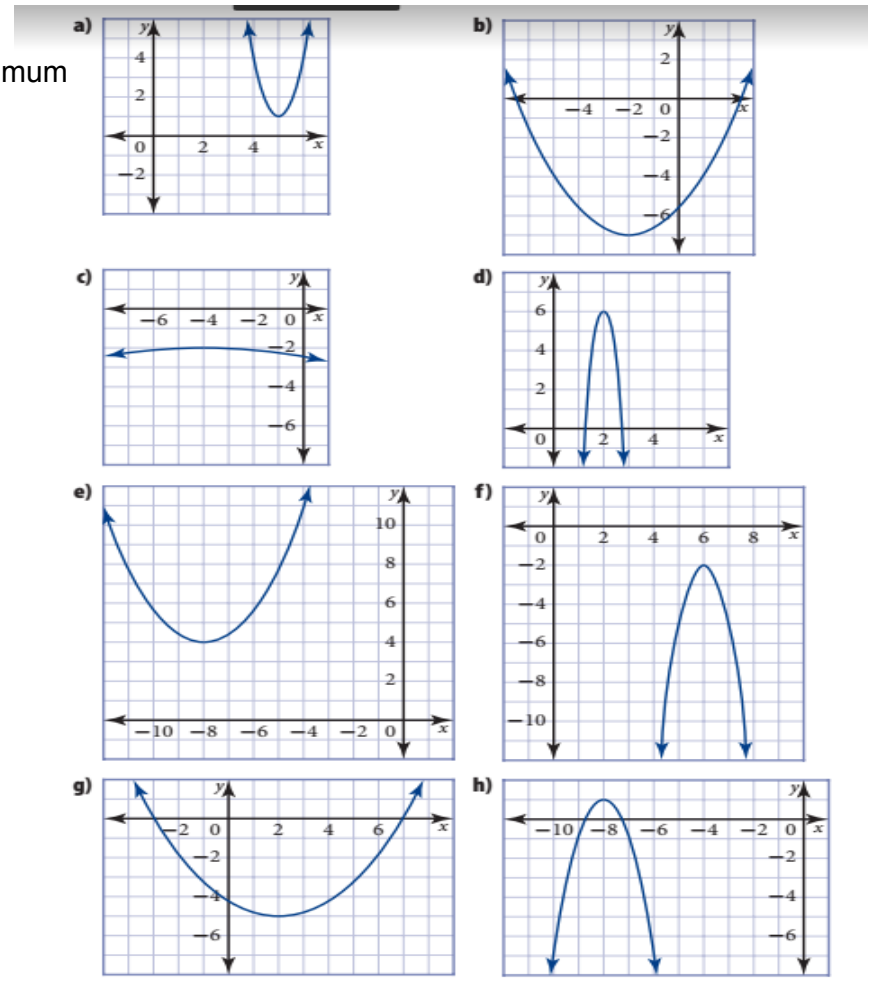
The maximum/~~minimum~~ value is 13 and it occurs when $x =$ 7

For each parabola, determine:

a) if there is a maximum or a minimum

b) what is the max/min value

c) when does it occur



For each parabola, determine:

a) if there is a maximum or a minimum

b) what is the max/min value

c) when does it occur

a) $y = 2(x - 3)^2 + 12$

b) $y = -0.5(x - 10)^2 - 1$

c) $y = -7(x + 4)^2 - 8$

d) $y = -(x + 20)^2 - 5$

e) $y = 0.5(x - 11)^2 - 3$

f) $y = 8(x + 2)^2 + 9$

g) $y = -0.5(x + 6)^2 + 7$

h) $y = 2(x - 8)^2 + 2$

i) $y = 7.5(x + 2)^2 - 1$

j) $y = -0.8(x - 4)^2 + 6$

If the equation of the parabola is in standard form or intercept form, we must locate the vertex by first locating the axis of symmetry, as we have done before.

Ex. 2 $y = -2x^2 - 8x + 24$

a) Does this relation have a maximum or a minimum value? Maximum
How do you know? "a" is negative ($a = -2$) ↪ ∴ opens downward

b) Find the zeroes of the relation.

$y = -2x^2 - 8x + 24$
$= -2(x^2 + 4x - 12)$
$0 = -2(x - 2)(x + 6)$
$x = 2$ $\quad x = -6$

c) What is the equation of the axis of symmetry?

$-1 + 12$
$-a + b$
$-3 + 4$

Ans: $x = \frac{2 + (-6)}{2}$
$= \frac{-4}{2}$
$x = -2$

d) Find the maximum/minimum value by substituting the x-value of the axis of symmetry into the equation of the quadratic.

$y = -2(x - 2)(x + 6)$
$= -2(-2 - 2)(-2 + 6)$
$= -2(-4)(4)$
$= 32$

The maximum value is 32,

and it occurs when $x =$ -2 .

Entertainment:

Now, on a piece of lined paper, for each of the following parabolas:
i) determine if there is a maximum or a minimum and
ii) state the max/min value and when it occurs.

a) $y = 2(x - 4)^2 + 1$

b) $y = -(x + 2)^2 + 21$

c) $y = -3(x + 5)^2 + 14$

d) $y = (x - 16)^2 - 35$

e) $y = x^2 - 6x + 5$

f) $y = -x^2 - 4x - 4$

g) $y = 3x^2 - 21x + 30$

h) $y = -4x^2 - 8x - 4$