

## 7.2 Compound Angle Formulas (Day 1)



"By the end of next class' lesson:  
I can derive and apply all compound angle formulas.  
I can apply what I have learned in unfamiliar settings."

### Trig Identities from last class...

#### Odd and Even

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

(see summary on p. 391)

#### Cofunction

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

(see summary on p. 392)

### On a circle with radius $r$ ...

$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

"SYR CXR TYX"

#### Recall:

##### Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

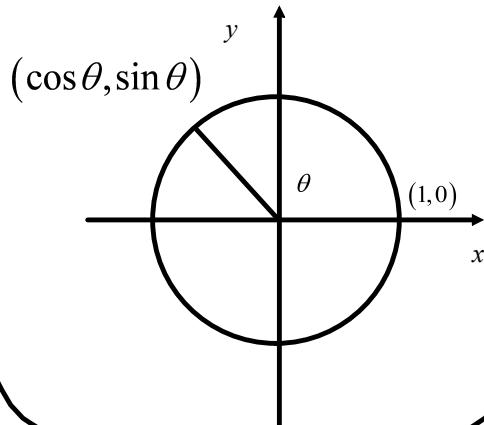
##### Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

##### Pythagorean Identities

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1 \\ \tan^2\theta + 1 &= \sec^2\theta \\ 1 + \cot^2\theta &= \csc^2\theta\end{aligned}$$

This means if  $r = 1$  then  
 $\sin\theta = y, \quad \cos\theta = x$



A sneak-preview of the  
Compound Angle Identities:

#### Addition Formulas

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

#### Subtraction Formulas

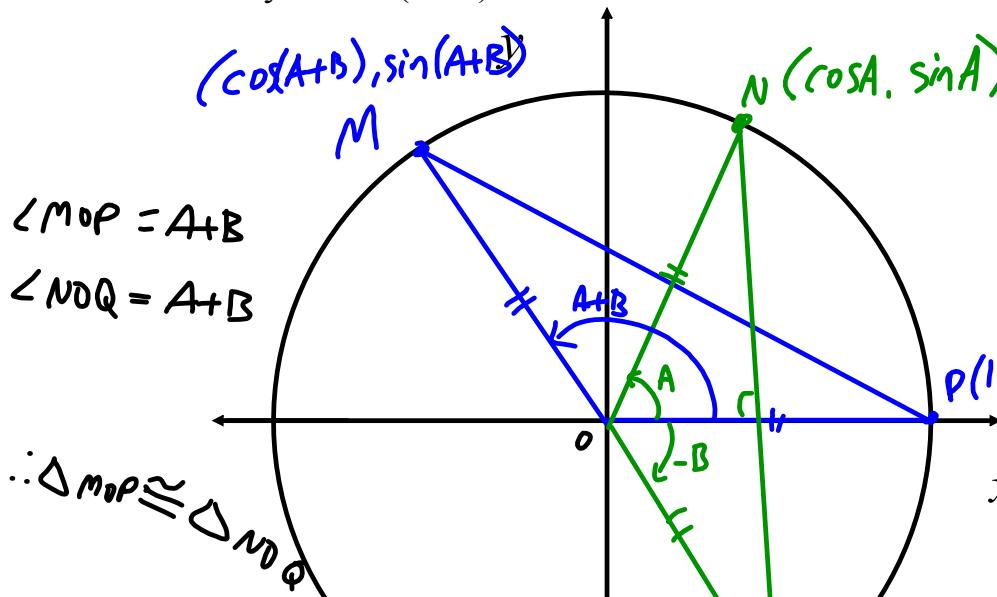
$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

A compound angle is an angle that is created by adding or subtracting two or more angles.

Create an identity for  $\cos(a + b)$



$$\begin{aligned} l &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(x + y)^2} \\ &= \sqrt{x^2 + 2xy + y^2} \end{aligned}$$

$$\text{square both sides } \therefore |MP| = |NQ|$$

$$\sqrt{(\cos(A+B) - 1)^2 + (\sin(A+B) - 0)^2} = \sqrt{(\cos A - \cos(-B))^2 + (\sin A - \sin(-B))^2}$$

$$\begin{aligned} \cancel{\cos^2(A+B)} - 2\cos(A+B) + 1 + \cancel{\sin^2(A+B)} &= \cancel{\cos^2 A} - 2\cos A \cos(-B) + \cancel{\cos^2(-B)} \\ 1 + 1 - 2\cos(A+B) &= 1 + 1 - 2\cos A \cos(-B) - 2\sin A \sin(-B) \end{aligned}$$

$$\cancel{-2\cos(A+B)} = \cancel{-2\cos A \cos B} - \cancel{2\sin A \sin(-B)}$$

$$\frac{-2\cos(A+B)}{-2} = \frac{-2\cos A \cos B + 2\sin A \sin B}{-2}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\text{Also: } \cos(a-b) = \cos a \cos b + \sin a \sin b$$

Now let's create an identity for  $\sin(a+b)$

$$\text{Recall: } \sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\begin{aligned}\sin(a+b) &= \cos\left(\frac{\pi}{2} - (a+b)\right) \\ &= \cos\left(\frac{\pi}{2} - a - b\right) \\ &= \cos\left(\frac{\pi}{2} - a\right) - b \\ &= \cos\left(\frac{\pi}{2} - a\right) \cos b + \sin\left(\frac{\pi}{2} - a\right) \sin b \\ &= \cos\left(\frac{\pi}{2} - a\right) \cos b + \sin\left(\frac{\pi}{2} - a\right) \sin b \\ &= \sin a \cos b + \cos a \sin b\end{aligned}$$

$$\therefore \sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\text{Also } \sin(a-b) = \sin a \cos b - \cos a \sin b$$

Ex. 1: Simplify to an exact value:

$$\cos \frac{5\pi}{12} \cos \frac{7\pi}{12} + \sin \frac{5\pi}{12} \sin \frac{7\pi}{12}$$

$$\begin{aligned}&= \cos\left(\frac{5\pi}{12} - \frac{7\pi}{12}\right) \\&= \cos\left(-\frac{2\pi}{12}\right) \\&= \cos\left(-\frac{\pi}{6}\right) \\&= \cos\left(\frac{\pi}{6}\right) \\&= \frac{\sqrt{3}}{2}\end{aligned}$$

Ex. 2: Find the exact value of  $\sin \frac{\pi}{12}$

$$\begin{aligned}\sin \frac{\pi}{12} &= \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) \\&= \sin \frac{\pi}{4} \cos \frac{\pi}{6} - \cos \frac{\pi}{4} \sin \frac{\pi}{6} \\&= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\&= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \\&= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

$\frac{\pi}{4} - \frac{\pi}{6}$
$= \frac{3\pi}{12} - \frac{2\pi}{12}$
$= \frac{\pi}{12}$

Entertainment...

pp.400-401 #1, 2b, 3, 4ade, 5abd, 6ace, 7ace, 8cd, 11, 16