7.4 Proving Trigonometric Identities

"By the end of next class:
I can prove any identity using previously established identities.
To disprove a claim, I understand that I only require a counterexample to it.
I can apply what I have learned in unfamiliar settings."

Identities Based on Definitions

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Cofunction Identities

$$\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\cos \theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan \theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

Identities Derived from Relationships

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Double Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Addition and Subtraction Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Odd and Even Function Identities

$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

Ex. 1: Prove $\dfrac{1}{1+\cos\theta}+\dfrac{1}{1-\cos\theta}=\dfrac{2}{\sin^2\theta}$

Ex. 2: Prove that $\cos 2x = 2\cos x$ is not an identity.

Ex. 3: Use a compound angle identity to prove that

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

Today's Work:pp. 417-418 #1, 5ac, 8 ,9abc, 17
Next class: p. 418 #10abce, 11bdgjl

The textbook answer section for 7.4 is poorly written.
For example, all identity proofs should always have a LS and RS chart; the answer section does not do this.