

7.4 Proving Trigonometric Identities



"By the end of next class:

I can prove any identity using previously established identities.

To disprove a claim, I understand that I only require a counterexample to it.

I can apply what I have learned in unfamiliar settings."

Identities Based on Definitions

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Cofunction Identities

$$\sin \theta = \cos \left(\frac{\pi}{2} - \theta \right)$$

$$\cos \theta = \sin \left(\frac{\pi}{2} - \theta \right)$$

$$\tan \theta = \cot \left(\frac{\pi}{2} - \theta \right)$$

Identities Derived from Relationships

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Double Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Addition and Subtraction Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Odd and Even Function Identities

$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

Ex. 1: Prove $\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = \frac{2}{\sin^2\theta}$

$$\begin{aligned}
 LS &= \frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} & LCD = ((1+\cos\theta)(1-\cos\theta)) \\
 &= \frac{1}{1+\cos\theta} \cdot \frac{1-\cos\theta}{1-\cos\theta} + \frac{1}{1-\cos\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta} \\
 &= \frac{1-\cos\theta + 1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)} \\
 &= \frac{2}{1-\cos^2\theta} \\
 &= \frac{2}{\sin^2\theta} \\
 &= RS \\
 \therefore QED.
 \end{aligned}$$

Ex. 2: Prove that $\cos 2x = 2\cos x$ is not an identity.

$$\text{if } x = \frac{\pi}{2}$$

$$\begin{aligned}
 LS &= \cos 2x & RS &= 2\cos x \\
 &= \cos 2\left(\frac{\pi}{2}\right) & &= 2\cos\left(\frac{\pi}{2}\right) \\
 &= \cos \pi & &= 2(0) \\
 &= -1 & &= 0
 \end{aligned}$$

$$\therefore LS \neq RS$$

$\therefore \cos 2x = 2\cos x$ is NOT an identity.

Ex. 3: Use a compound angle identity to prove that

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\text{LS} = \sin\left(\frac{\pi}{2} - \theta\right)$$

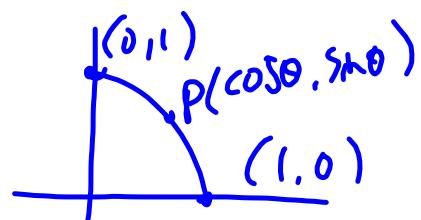
$$= \sin \frac{\pi}{2} \cos \theta - \cos \frac{\pi}{2} \sin \theta$$

$$= (1)\cos \theta - (0)\sin \theta$$

$$= \cos \theta$$

$$= \text{RS}$$

$\therefore \text{QED}$



Today's Work: pp. 417-418 #1, 5ac, 8, 9abc, 17

Next class: p. 418 #10abce, 11bdgjl

The textbook answer section for 7.4 is poorly written.

For example, all identity proofs should always have a LS and RS chart; the answer section does not do this.