7.4 Proving Trigonometric Identities (Day 2)

"By the end of next class:
I can prove any identity using previously established identities.
To disprove a claim, I understand that I only require a counterexample to it.
I can apply what I have learned in unfamiliar settings."

Today's Work: p. 418 #10abce, 11bdgjl

The textbook answer section for 7.4 is poorly written.
For example, all identity proofs should always have a LS and RS chart;
the answer section does not do this.

Identities Based on Definitions

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Cofunction Identities

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

Identities Derived from Relationships

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Double Angle Formulas

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Addition and Subtraction Formulas

$$\sin(x + y) = \sin x\cos y + \cos x\sin y$$

$$\sin(x - y) = \sin x\cos y - \cos x\sin y$$

$$\cos(x + y) = \cos x\cos y - \sin x\sin y$$

$$\cos(x - y) = \cos x\cos y + \sin x\sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$$

Odd and Even Function Identities

$$\cos(-\theta) = \cos\theta$$

$$\sin(-\theta) = -\sin\theta$$

Last day's Work:pp. 417-418 #1, 5ac, 8 ,9abc, 17

p. 418

17. Express $8\cos^4 x$ in the form $a\cos 4x + b\cos 2x + c$. State the values of the constants a, b, and c.

$= 8\cos^4 x$

$= 2(2\cos^2 x)(2\cos^2 x)$

$= 2(\cos 2x + 1)(\cos 2x + 1)$

$= 2(\cos^2 2x + 2\cos 2x + 1)$

$= 2\cos^2 2x + 4\cos 2x + 2$

$= \cos 4x + 1 + 4\cos 2x + 2$

$= \cos 4x + 4\cos 2x + 3$

$\therefore a = 1 \quad b = 4 \quad c = 3$

$a\cos 4x + b\cos 2x + c$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x + 1 = 2\cos^2 x$

if 4

$2\cos^2 x = \cos 2x + 1$

$2\cos^2 2x = \cos 4x + 1$