7.5 Solving Linear Trigonometric Equations

"I can solve for the unknown angle(s) in any linear trigonometric equation.
I realize that I may need to apply previously established identities to do so.
I can apply what I have learned in unfamiliar settings."

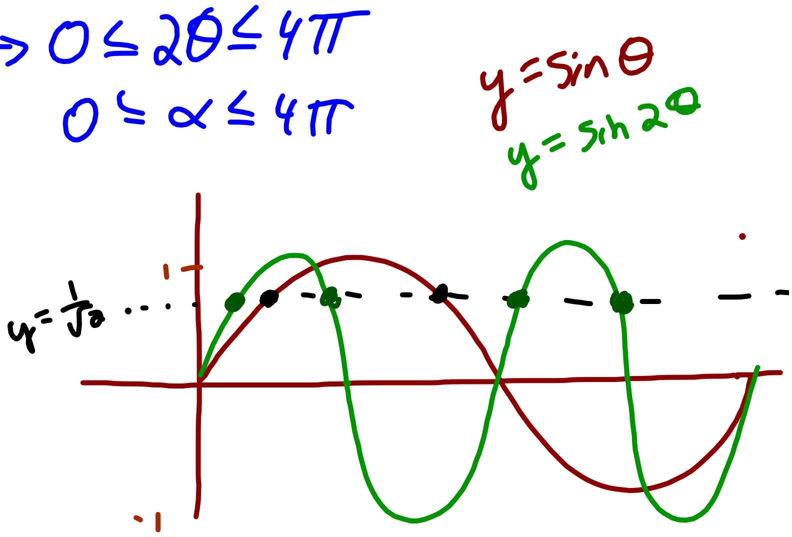
Ex. 1: Solve: $\sin 2\theta = \frac{1}{\sqrt{2}}$, $0 \le \theta \le 2\pi$

Let $2\theta = \alpha$

$\hookrightarrow 0 \le 2\theta \le 4\pi$

$0 \le \alpha \le 4\pi$

$\therefore \sin\alpha = \frac{1}{\sqrt{2}}$

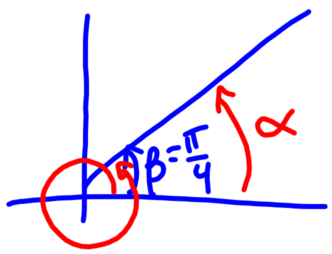


raa: $\sin\beta = \frac{1}{\sqrt{2}}$

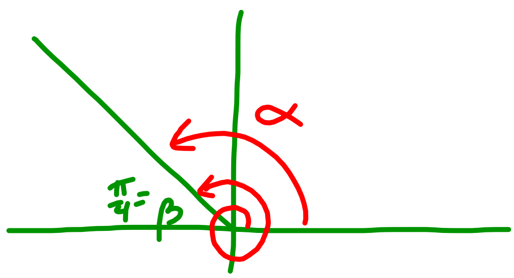
$\beta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\pi}{4}$

in QI



in QII



$\alpha = \frac{\pi}{4}$ or $\alpha = 2\pi + \frac{\pi}{4}$

$= \frac{9\pi}{4}$

$\alpha = \frac{3\pi}{4}$ or $\alpha = 3\pi - \frac{\pi}{4}$

$= \frac{11\pi}{4}$

$2\theta = \frac{\pi}{4}$ $\quad 2\theta = \frac{9\pi}{4}$ $\quad 2\theta = \frac{3\pi}{4}$ $\quad 2\theta = \frac{11\pi}{4}$

$\theta = \frac{\pi}{8}$ $\quad \theta = \frac{9\pi}{8}$ $\quad \theta = \frac{3\pi}{8}$ $\quad \theta = \frac{11\pi}{8}$

$\therefore \theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

Ex. 2: Solve over the interval given. State exact answers in radians.

$$\cos^2\theta - \sin^2\theta = \sqrt{3}\sin 2\theta,\ 0 \le \theta \le 2\pi$$

$$\frac{\cos 2\theta}{\cos 2\theta} = \frac{\sqrt{3}\sin 2\theta}{\cos 2\theta} \qquad \rightarrow \quad 0 \le 2\theta \le 4\pi$$

$$0 \le \alpha \le 4\pi$$

$$1 = \sqrt{3}\tan 2\theta$$

$$\frac{1}{\sqrt{3}} = \tan 2\theta$$

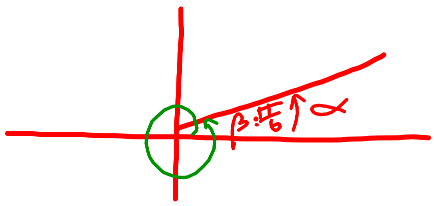
Let $2\theta = \alpha$

$$\therefore \tan\alpha = \frac{1}{\sqrt{3}}$$

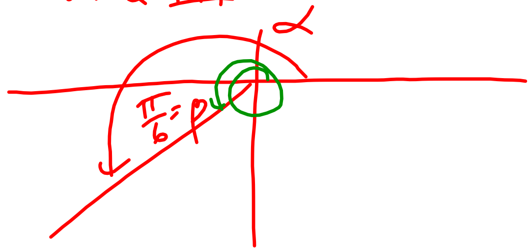
raa: $\tan\beta = \frac{1}{\sqrt{3}}$

$$\beta = \frac{\pi}{6}$$

in QI



in QIII



$\alpha = \frac{\pi}{6}$ or $\alpha = 2\pi + \frac{\pi}{6} = \frac{13\pi}{6}$

$\alpha = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ or $\alpha = 3\pi + \frac{\pi}{6} = \frac{19\pi}{6}$

$2\theta = \frac{\pi}{6}$, $\theta = \frac{\pi}{12}$

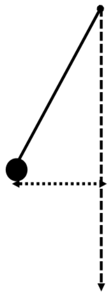
$2\theta = \frac{13\pi}{6}$, $\theta = \frac{13\pi}{12}$

$2\theta = \frac{7\pi}{6}$, $\theta = \frac{7\pi}{12}$

$2\theta = \frac{19\pi}{6}$, $\theta = \frac{19\pi}{12}$

$$\therefore \theta = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$$

Ex. 3: A pendulum swings. The displacement from centre in centimetres (d) over time in seconds (t) is given by:



$$d = -2\cos\frac{\pi}{2}t$$

Determine the first two time values when the horizontal distance from centre is 1.3 cm, each rounded to the nearest hundredth.

$1.3 = -2\cos\frac{\pi}{2}t$

$-0.65 = \cos\frac{\pi}{2}t$

Let $\frac{\pi}{2}t = \alpha$

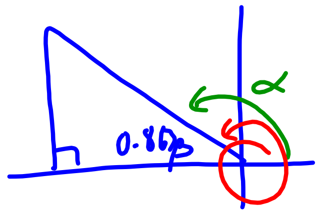
$\cos\alpha = -0.65$

raa $\cos\beta = 0.65$

$\beta = \cos^{-1}(0.65)$

$\doteq 0.863$ radians

in QII



$\therefore \alpha = \pi - \beta$ or $\alpha = 3\pi - \beta$

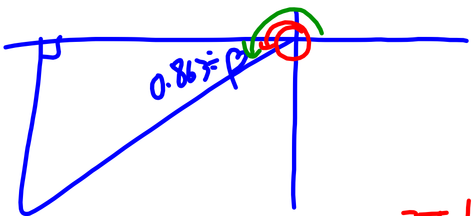
$\doteq 2.278$ $\doteq 8.561$

$\frac{\pi}{2}t \doteq 2.278$

$t \doteq \frac{2}{\pi}(2.278)$

$\doteq 1.450$

in QIII



$\alpha = \pi + \beta$ or $\alpha = 3\pi + \beta$

$\doteq 4.004$ $\doteq 10.287$

$\frac{\pi}{2}t \doteq 4.004$

$t \doteq \frac{2}{\pi} \cdot 4.004$

$\doteq 2.549$

$\doteq 2.55$

$\therefore$ the first 2 time values when the displacement is 1.3 cm are 1.45 s and 2.55 s.

Entertainment pp.426-428 #3, 6de, 7de, 9de, 10ef, 11*, 13

*Hint: produce a sketch first