8.1 Exploring the Logarithmic Function

The **logarithmic function** $y = \log_b x$ is the *inverse* of the exponential function $y = b^x$.

Note:

- Both relations above are functions;
- The x-axis is the HA for the exponential function

The y-axis is the VA for the logarithmic (log) function

- The y-intercept of the exponential function is 1, while

the x-intercept of the log function is 1

- The range of the exponential function is $\{y \quad \mathbf{R} \mid y > 0\}$,

so the domain of the log function is $\{x \quad \mathbf{R} \mid x > 0\}$.

- The domain of the exponential function is $\{x \quad \mathbf{R}\}$,

so the range of the log function is $\{y \quad \mathbf{R}\}$.

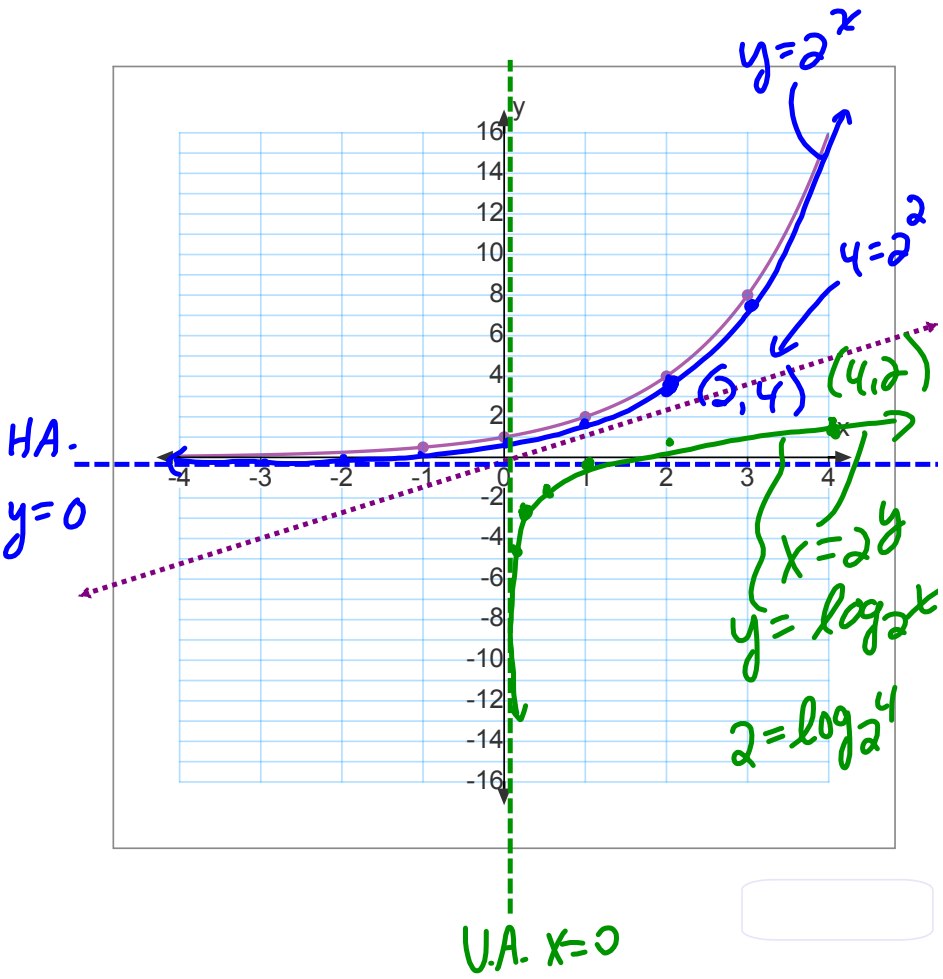
- The base, b, where $b > 0$, and $b \neq 1$.

Ex. 1:

Complete a table of values for $y = 2^x$ using integer values for x.
Confirm the properties described above, after graphing its inverse.

$y = 2^x$

x	y
-3	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
-2	$\frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	1
1	2
2	4
3	8



Ex. 2:

Complete a table of values for $y = \left(\frac{1}{2}\right)^x$ using integer values for x.

Note: $y = \left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$

Confirm the properties described on the first page after graphing the inverse.

$$y = \left(\frac{1}{2}\right)^x$$

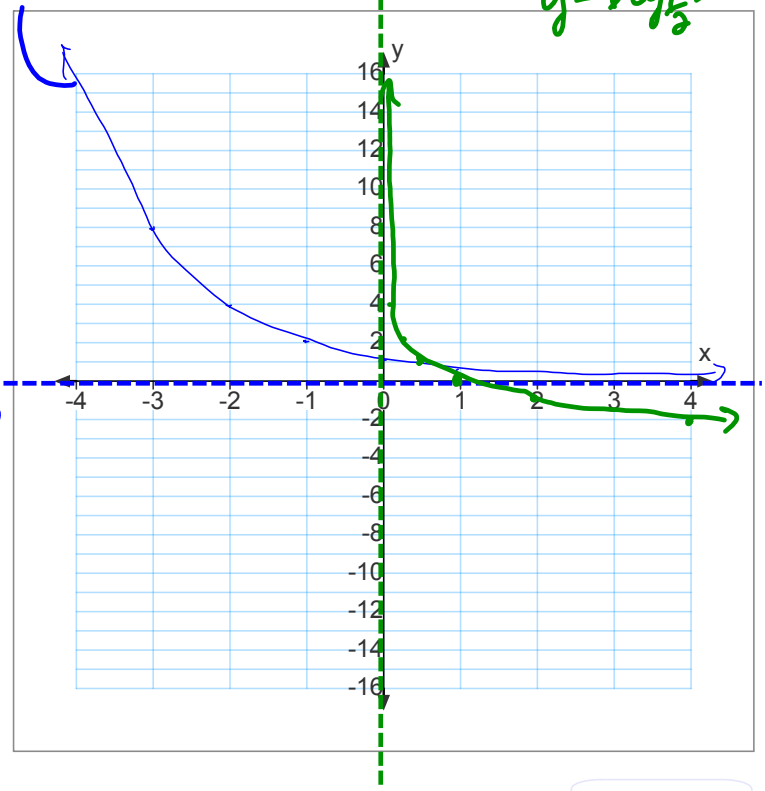
x	y
-3	$\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$

$y = \left(\frac{1}{2}\right)^x$

VA: $x = 0$ $x = \left(\frac{1}{2}\right)^y$

$y = \log_{\frac{1}{2}} x$

HA $y = 0$



Note: Given $y = b^x$,

When $b > 1$ Ex. $y = 2^x$, $b = 2$ exponential growth

logarithmic growth

When $0 < b < 1$

Ex. $b = \frac{1}{2}$

$\therefore y = \left(\frac{1}{2}\right)^x$ exponential decay

logarithmic decay

It makes sense that if the log function is the inverse of the exponential function, then ...

Start with $y = b^x$ $\quad x \leftrightarrow y \quad$ Inverse $x = b^y$

$y = \log_b x$

Ex. 3: Without using a calculator, evaluate:

a) $\log_5 25 = 2$ b) $\log_{55} 1 = 0$ c) $\log_2\left(\frac{1}{4}\right)$ d) $\log_{10} 1000$

$y = \log_5 5^2$ $\quad = -2$ $\quad = 3$

"I understand that a logarithmic function is the inverse of an exponential function.
Also, I know their associated properties.
Finally, I can evaluate simple logarithms without a calculator."

Entertainment: p.451 #1ac, 2i and ii for 1a and 1c, 3ac, 4 to 9*, 10, 11
*Note: for #9c the answer is 3.