8.4 Laws of Logarithms

"I can prove any logarithmic property and any logarithmic law studied. Moreover, I know when they apply in a given mathematical scenario. I can apply what I have learned in unfamiliar settings."

Recall:

Properties of Logarithms:

i) $\log_a 1 = 0$ ii) $\log_a a = 1$ iii) $\log_a a^x = x$ iv) $a^{\log_a x} = x$

Exponent Laws:

multiplication division power

1) $a^m \cdot a^n = a^{m+n}$ 2) $a^m \div a^n = a^{m-n}$ 3) $(a^m)^n = a^{m \times n}$

Ex. 1: Prove $\log_a mn = \log_a m + \log_a n$, where a, m, n are positive, and $a \neq 1$.

let $m = a^x$, $n = a^y$ $\rightarrow$ if $m = a^x$ if $a^y = n$

$\log_a m = x$ $\log_a n = y$

$LS = \log_a mn$

$= \log_a (a^x)(a^y)$

$= \log_a (a^{x+y})$

$= x + y$

$= \log_a m + \log_a n$

$= RS$

$\therefore$ QED!

Ex. 2: Prove $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$, where a, m, n are positive, and $a \neq 1$.

let $m = a^x$, $n = a^y$ $\rightarrow$ if $m = a^x$ if $a^y = n$

$\log_a m = x$ $\log_a n = y$

$LS = \log_a \left(\frac{m}{n}\right)$

$= \log_a \left(\frac{a^x}{a^y}\right)$

$= \log_a a^{x-y}$

$= x - y$

$= \log_a m - \log_a n$

$= RS$

$\therefore$ QED!

Ex. 3: Prove $\log_a(m^n) = n\log_a m$, where a, m, n are positive, and $a \neq 1$.

let $m = a^x$ $\rightarrow$ if $m = a^x$, $\log_a m = x$

$LS = \log_a m^n$

$= \log_a (a^x)^n$

$= \log_a a^{xn}$

$= xn$

$= nx$

$= n\log_a m$

$= RS$

$\therefore$ QED

Ex. 4: Without a calculator, express as a single logarithm, then evaluate.

a) $\log_4 192 - \log_4 3$

$= \log_4\left(\frac{192}{3}\right)$

$= \log_4 64$

$= 3$

b) $\log_8 6 - \log_8 3 + \log_8 4$

$= \log_8 (6 \div 3 \times 4)$

$= \log_8 8$

$= 1$

c) $2\log_4 8$

$= \log_4 8^2$

$= \log_4 64$

$= 3$

Ex. 5: Using algebraic reasoning, compare $f(x) = \log(100x)$ and $g(x) = 2 + \log x$

$f(x) = \log(100 \cdot x)$

$= \log 100 + \log x$

$= 2 + \log x$

$= g(x)$

$\therefore$ same function.

Ex. 6: Evaluate without a calculator: $3^{-\frac{1}{2}\log_3 49}$

$= 3^{\log_3 49^{-\frac{1}{2}}}$

$= 49^{-\frac{1}{2}}$

$= \frac{1}{49^{\frac{1}{2}}}$

$= \frac{1}{\sqrt{49}}$

$= \frac{1}{7}$

Entertainment

pp. 475-476 #1ade, 2ade, 3ade, 4ade, 5, 6ade, 7ad, 9ade, 10a, 11ade, 13*, 16

*answer is: vertical stretch by a factor of 3, and vertical translation up 3 units